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The aim is to meet the needs of the average student in mathematics. Modern concepts based on the findings of experimental programs are included. Individual differences among students are considered, and appropriate mathematical sequences, developed. Courses of study in mathematics are listed for the five "lanes" of the Chicago school system, grades 9-12 (highest achievement, above-average achievement, average achievement, below-average achievement and low achievement in mathematics). Subjects in this guide include First Year Algebra, Plane Geometry, Algebra 3 and 4, Trigonometry, Practical Trig and Solid Geometry, each subdivided into units of learning with objectives, teaching suggestions, related activities, and time allotments. An appendix contains articles on the development of our number system and sets; a detailed bibliography, including periodicals, films and books, is listed.

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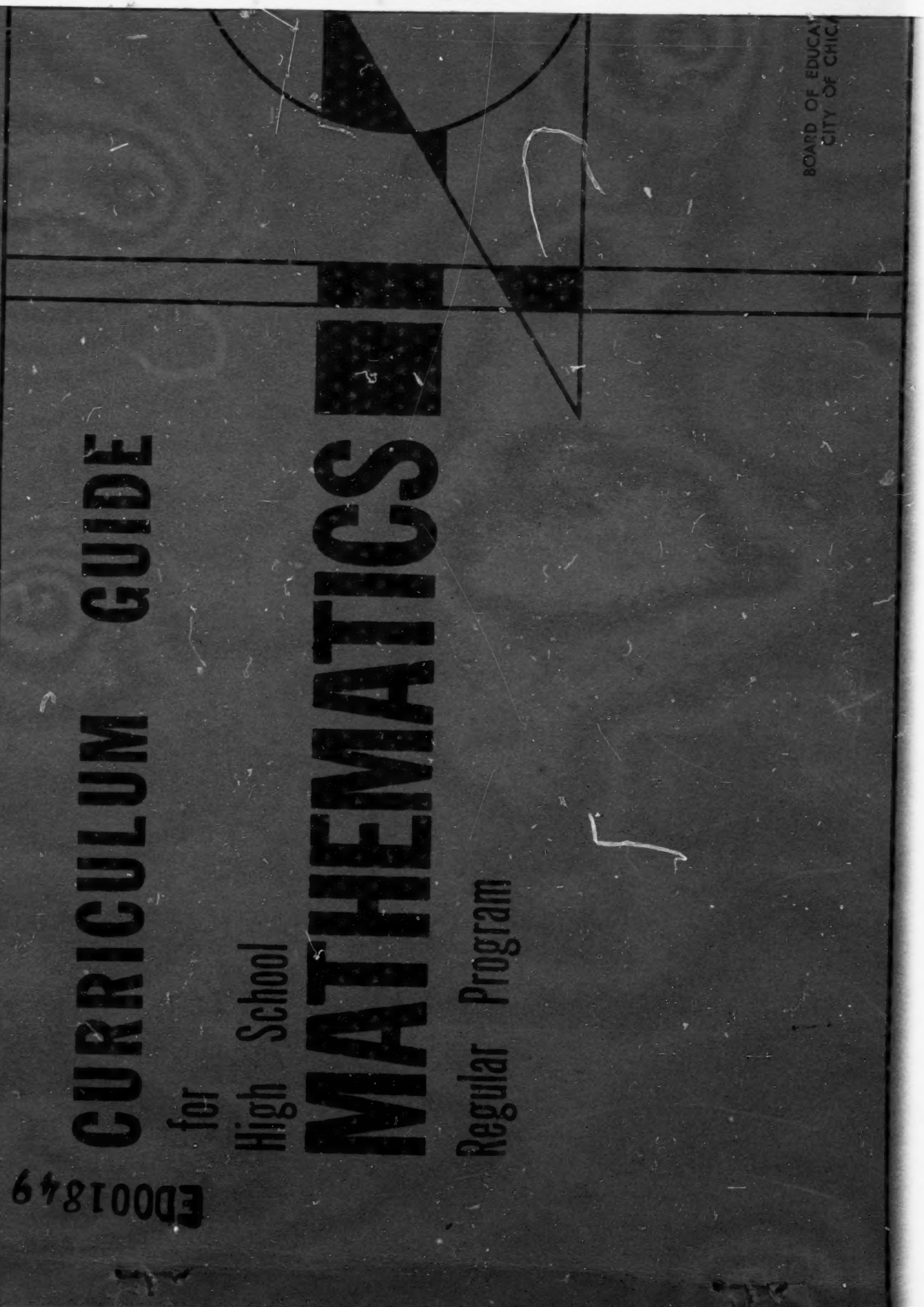

BOARD OF EDUCA
CITY OF CHIC

CURRICULUM GUIDE

for

High School

MATHEMATICS

Regular Program

ED018449

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HIGH SCHOOL MATHEMATICS

REGULAR PROGRAM

Copyright 1962
Board of Education
of the
City of Chicago
Chicago, Illinois
Revised 1964

BENJAMIN C. WILLIS

General Superintendent of Schools

BOARD OF EDUCATION
CITY OF CHICAGO
1962

REVISED 1964

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MAY 1965
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FOREWORD

Meeting the needs of the average, or able, students in mathematics is a function of great importance in the educational program at the secondary school level. This large group of young people will need more mathematics in their daily lives than is needed by the adults of today.

There is a fast growing national need for people skilled in various branches of mathematics. This fact provides a compelling reason for an expanding mathematics curriculum. The demand for mathematically trained men and women stems from business, industry, and government, as well as from science and engineering. The applications of mathematics in electronics, in the use of computing machines, in other phases of automation, and in industrial research have opened up new opportunities for persons trained in mathematics. The course offerings in this guide were selected to provide for these increased needs.

Modern concepts are included in this guide based on the findings of experimental programs in modern mathematics which were conducted in the Chicago public schools and in other schools throughout the nation. These concepts help to update the mathematics program.

The guide takes into consideration the individual differences of students and makes adequate provision for these differences. The "Related Activities" and "References" given for each unit provide motivation as well as meaningful and satisfying experiences for these able students.

BENJAMIN C. WILLIS

General Superintendent of Schools

ACKNOWLEDGMENT

DEPARTMENT OF INSTRUCTION

James H. Smith
Deputy Superintendent

DEPARTMENT OF INSTRUCTION

Thaddeus J. Lubera
Associate Superintendent

DEPARTMENT OF CURRICULUM DEVELOPMENT

Evelyn F. Carlson
Associate Superintendent

DEPARTMENT OF OPERATION SERVICES

Edwin A. Lederer
Associate Superintendent

DEPARTMENT OF ADMINISTRATION

Milton J. Cohler
Associate Superintendent

DEPARTMENT OF HIGHER EDUCATION

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HIGH SCHOOL MATHEMATICS EVALUATION COMMITTEE

Herbert W. Lehmann, Director, Division of Work Experience and Post High School Guidance, Chairman
Lucile B. Jautz, Mathematics Consultant, Department of Curriculum Development
Mildred Levin, Teacher, Clinton Elementary School
Charles McCarthy, Assistant Principal, Hyde Park High School
Henry A. Patin, Associate Professor of Education, Chicago Teachers College South
Lucy Periolat, Head Teacher, West Pullman Branch, Fenger High School
Edward J. Prokop, Assistant Principal, Froebel Branch, Harrison High School
Margaret Rubly, Teacher, Lake View High School
Richard H. Sanders, Vice-Chairman, Board of Examiners
Raymond R. Wallace, Mathematics Consultant, Department of Curriculum Development

Grateful appreciation is expressed to the teachers who provided thoughtful and excellent comments about the field test copies for the improvement of the guide for mathematics, and to Bernice H. Antoine, Harry S. Clare, Lucy Periolat, and Raymond R. Wallace for their help in the preparation of this guide.

We are deeply indebted to Dr. William Meyers, University of Chicago; Dr. Harriet Montague, New York State University; and Dr. John Wagner, Michigan State University, for their critical reading of these materials.

Cover Design by Homer Freese

Students may transfer from one lane to another with the approval of the principal.

LANE	1	2	3	4	5
GRADE LEVEL	ADVANCED PLACEMENT HIGHEST ACHIEVEMENT IN MATHEMATICS	ABOVE-AVERAGE ACHIEVEMENT IN MATHEMATICS	AVERAGE ACHIEVEMENT IN MATHEMATICS	BELOW-AVERAGE ACHIEVEMENT IN MATHEMATICS	LOW ACHIEVEMENT IN MATHEMATICS
9	Algebra I (Honors) Algebra II (Honors) Algebra III (Honors)	Algebra I (Honors) Algebra II (Honors)	Algebra I Algebra II	Essential Mathematics I Essential Mathematics II	Basic Mathematics I Basic Mathematics II
10	Plane and Solid Geometry	Plane Geometry I (Honors) Plane Geometry II (Honors)	Plane Geometry I Plane Geometry II	Essential Mathematics III Essential Mathematics IV	
11	Trigonometry (Honors) College Algebra	Algebra III (Honors) Trigonometry (Honors)	Algebra III Trigonometry Practical Trigonometry*		
12	Analytic Geometry and Calculus*	College Algebra* Analytic Geometry Modern Algebra Elementary Functions	Algebra IV Solid Geometry		
NOTES	Students in this sequence may complete five years of mathematics in four years. They may receive credit in freshman college mathematics through examination. *Required to qualify for the Advanced Placement Examination.	The honors sequence provides for a deep and rich understanding of mathematical concepts. The modern approach is an integral part of the courses in this sequence. *A Choice of four electives is provided.	This is the regular college preparatory sequence. Concepts from modern mathematics are included. Trigonometry, Solid Geometry, and Algebra IV may be offered in the order preferred by the local school. *Practical Trigonometry is not a college preparatory course.	In general, this sequence is for students who have at least 6.0 grade achievement in arithmetic and do not plan to go to college. It is also for those who need more training before taking the college preparatory sequence. Emphasis is on the mathematics of daily living.	This sequence is for students with low achievement in mathematics, below 6.0 on a standardized test. Emphasis is on the basic skills and concepts of arithmetic, particularly fractions, decimals, and percentage.

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ALGEBRA I

I. INTRODUCTION

A. Development of the Number System

1. Natural numbers
2. Integers
3. Rational numbers
4. Irrational numbers
5. Real numbers

B. The Number Line

1. Natural numbers and zero
2. Positive rational numbers
3. Positive irrational numbers

Objective

To understand that the history of the development of our number system is a unifying element in mathematics

Teaching Suggestions

Show that the development of the number system and its expansion parallels closely the history of our civilization. Whenever possible, give specific instances of this relationship.

The number line is used to teach arithmetic concepts in the elementary schools. It should be reviewed at this time.

The development of the number system is expanded in the appendix for the use of the teacher. This number system cannot be considered final even at the present time. New number systems are constructed as the need arises.

Properties of real numbers are also presented in the appendix.

This unit gives the necessary background for the unit on sets. It presents the historical information which adds interest to the classwork.

Related Activities

The teacher may assign individual student reports based on library research, on the following topics:
the need of counting by primitive man
the Hindu-Arabic system of notation
the Babylonian number system

FIRST YEAR ALGEBRA

Courses of study for the gifted and mathematically-talented students have received much attention in the past few years. During this time experiments with "modern mathematics" have been carried on throughout the nation. These experiments seem to substantiate the claim that the modern approach makes the subject more meaningful. In this outline for First Year Algebra for the average student some of the concepts of modern mathematics are included. This is done in compliance with the recommendations of the High School Mathematics Evaluation Committee.

Articles on modern mathematics have appeared frequently in daily papers and magazines. Some of these have confused the public with their general statements which have little meaning for the average person.

Modern concepts simplify mathematics. These make mathematics more precise and understandable and give a logical explanation of the common arithmetical skills. This is done by a different approach, by more meaningful symbols, and by a more precise vocabulary. Some of the symbols are not new but have a new meaning. The difference between the symbol and the object for which the symbol stands is emphasized. The concept of sets and more work with inequalities are suggested for use in this course for First Year Algebra.

The first two units "The Development of the Number System" and "Sets" are expanded in the appendix to make this material readily available.

Teachers are encouraged to try these units with their classes.

The first year course in algebra should prepare the student for the high school mathematics program. Some knowledge of the structure of the subject becomes a unifying factor. The structure forms the beginning for the logical development of mathematics.

This course of study is provided for a one year course looking forward to yearly promotion. However, the courses for Algebra I and II are clearly indicated.

the abacus
number systems to other bases
binary
ternary
quinary
duodecimal.

View films Base and Place, Electronic Computers and Applied Mathematics, and Symbols in Algebra.

References

Courant and Robbins. What Is Mathematics, pp. 1-9.

Hooper. Makers of Mathematics, pp. 3-16.

Logsdon. A Mathematician Explains, pp. 15-20.

National Council of Teachers of Mathematics. Insights into Modern Mathematics, pp. 1-16.

Suggested Time

One week

II. SETS

A. Basic Concepts of Sets and Symbols for Sets

1. Mathematical meaning of sets
2. Subsets
3. Universal set

Objectives

To acquire an understanding of some of the common terms and operations in set theory

To see that set theory increases understanding of the subject matter and the precision and generality of the subject

B. Operation with Sets

1. Diagrams
2. Intersection and union of sets
3. Complementary sets
4. Solution of problems with sets

Teaching Suggestions

Set theory is included in some of the newer textbooks. An expansion of this unit is found in the appendix on pages to . More problems are given in the references below. Since this unit is in the experimental stage, the use of this material is left to the discretion of the teacher.

Related Activities

The teacher may ask the students to prepare:
charts illustrating the meaning of sets
diagrams showing operations with sets
nonmathematical examples of sets
reports on the concepts of this unit, such as finite and infinite sets
reports of current publications on sets.

References

Aiken and Beseman. Modern Mathematics: Topics and Problems, pp. 1-38.

Johnson and Glenn. Sets, Sentences, and Operations, pp. 1-34.

Krickenberger and Pearson. An Introduction to Sets and the Structure of Algebra, pp. 1-20.

Nichols and Collins. Modern Elementary Algebra, pp. 1-91.

School Mathematics Study Group. Mathematics for High School: First Course in Algebra, Part I, pp. 1-93.

Woodward and McLennan. Elementary Concepts of Sets, pp. 1-24.

III. SENTENCES

A. Introduction to Sentences

1. True mathematical sentences
2. False mathematical sentences

B. Formulas

1. Common formulas for areas and volumes
2. Common formulas from business and industry
3. Translation of formulas into words
4. Translation of words into formulas

C. Open Sentences Involving Equalities

1. Variables and constants
2. Sentences of condition
3. Sentences of identity
4. Explanation of "roots" of an equation, "solution sets," and "truth sets"
5. Representation of roots on the number line
6. Solution of simple sentences by the axioms of division, multiplication, addition, and subtraction

D. Solution of Word Problems by Using Mathematical Sentences

Suggested Time

Two weeks

Objectives

To become familiar with algebraic operations where letters replace numbers

To learn how to solve simple equations and inequalities

Teaching Suggestions

If Unit II has been taught in detail, the solution of equations and inequalities should be taught by means of the modern concepts. If not, traditional methods should be used to teach the solution of simple equations. Inequalities should be introduced. Word problems may be used to add interest. Representation of roots on the number line will prepare the students for extension of the number line to include negative numbers.

Related Activities

The teacher may direct students to make reports on:
the history of the concepts in this unit
the meaning of the word "algebra."

View film Sentences and Solution Sets.

References

Aiken and Beseman. Modern Mathematics: Topics and Problems, pp. 10, 11, 16-22, 26-29.

E. Sentences Involving Inequalities

1. Symbols: $<$, less than; $\not<$, not less than; $>$, greater than; $\not>$, not greater than
2. Representation of inequalities on the number line as

$x < 5$, and $x > 7$

Breuer. Introduction to the Theory of Sets.

Commission on Mathematics of the College Entrance Examination Board. Sets, Relations, and Functions, pp. 3-9.

Johnson and Glenn. Sets, Sentences, and Operations, pp. 40-43.

Logsdon. A Mathematician Explains, pp. 51-54.

School Mathematics Study Group. Mathematics for High School: First Course in Algebra, Part I, pp. 94-118.

IV. POSITIVE AND NEGATIVE NUMBERS

Suggested Time

Three weeks

Objectives

To extend the use of the number line to include negative numbers

To understand the meaning of absolute value

To learn the four fundamental operations with signed numbers

A. The Real Number Line

1. Negative numbers, the opposite or inverse of positive numbers
2. Positive and negative numbers on the number line

B. Absolute Value

1. Definition
2. Symbol, "| |", read "the absolute value of"
3. Positive number

$|6| = 6$

Teaching Suggestions

The set of real numbers studied in Unit I should be reviewed. The representation of a number as a unique point on the line should be emphasized. The fact should be stressed frequently that there is a point on the line for every real number, rational or irrational. Opposites, absolute value, and order relations should be defined.

4. Zero

$|0| = 0$

5. Negative number

$|-2| = 2$

6. Geometric representation

Related Activities

The teacher may suggest student reports on:
history of negative numbers
daily papers and magazine articles using the symbols "+" and "-" to tell something about the numbers as Board of Trade and weather reports.

View film Formulas in Mathematics.

C. The Four Fundamental Operations with Positive and Negative Numbers (Numbers Only)

D. Evaluation of Formulas Using Positive and Negative Values of the Letters

E. Solution of Simple Sentences (Equations) with Both Positive and Negative Roots

References

Aiken and Beseman. Modern Mathematics: Topics and Problems, pp. 25-38.

Nichols and Collins. Modern Elementary Algebra, pp. 50-69.

School Mathematics Study Group. Mathematics for High School: First Course in Algebra, Part I, pp. 119-140.

Woodward and McLennan. Elementary Concepts of Sets, pp. 25-34.

Basic and auxiliary textbooks

Suggested Time

Five weeks

V. OPERATIONS OF ADDITION AND SUBTRACTION WITH SIGNED NUMBERS

A. Monomials with Integral, Fractional, and Decimal Coefficients

Objective

To develop the skills of addition and subtraction with signed numbers and with polynomials and the application of these skills to problem solving

B. Polynomials

1. Expressions with numeral coefficients
2. Expressions with literal coefficients

C. Removal and Insertion of Parentheses

D. Solution of Sentences (Equations) and Word Problems by the Use of the Axioms of Addition and Subtraction

Teaching Suggestions

This unit lends itself to a review of adding and subtracting fractions and decimals. Fractional and decimal coefficients may also be used in word problems.

All the basic textbooks have adequate exercises and problems for this unit. The first reference shows the presentation of this unit in a modern algebra textbook. The second reference given below shows how the School Mathematics Study Group presents these concepts. The third reference gives some practical applications of positive and negative numbers.

Related Activities

The teacher may ask the students to make individual reports on the following:

history of the use of symbols to represent unknowns
meaning of "root," "solution," "solution set," and "truth set"
early use and present-day use of grouping symbols.

References

Nichols and Collins. Modern Elementary Algebra, pp. 50-75.

School Mathematics Study Group. Mathematics for High School: First Course in Algebra, Part II, pp. 141, 163, 196-207.

Weeks and Adkins. First Course in Algebra, pp. 184-185.

Basic and auxiliary textbooks

Suggested Time

Two weeks

VI. OPERATIONS OF MULTIPLICATION AND DIVISION WITH SIGNED NUMBERS

A. Definition and Use of Exponents

B. Multiplication

1. Monomials by monomials
2. Polynomials by monomials
3. Polynomials by polynomials

C. Solution of Sentences (Equations) and Word Problems

D. Division

1. Monomials by monomials
2. Polynomials by monomials
3. Polynomials by polynomials

E. Solution of Sentences and Word Problems

F. Solution of Literal Sentences and Solution of Formulas by Changing the Subject

Objective

To develop the skills of multiplication and division with signed numbers and with polynomials and the application of these skills to problem solving

Teaching Suggestions

The basic textbooks have adequate explanations and exercises for this unit. The students should have mastery of the skills of the four fundamental operations and of the solution of simple equations. Review the concepts which the class has not mastered.

Related Activities

The following list of activities may be suggested by the teacher to the students for added interest:

puzzles solved by equations
scientific notation (use of exponents)
bulletin board display of algebraic graphs
discussion of trades and professions which require a knowledge of algebra
report on the early use of exponents
report on the development of multiplication and division, and the symbols for multiplication and division.

References

Bakst. Mathematical Puzzles and Pastimes.

Nichols and Collins. Modern Elementary Algebra, pp. 75-91.

Northrop. Riddles in Mathematics.

School Mathematics Study Group. Mathematics for High School: First Course in Algebra, Part II, pp. 145-181.

Basic and auxiliary textbooks

VII. SENTENCES (EQUATIONS) WITH TWO UNKNOWNS

A. Linear Relation Between Two Variables by Graphing

1. Representation of a point by an ordered pair of numbers (x-value and y-value or abscissa and ordinate)
2. Definition of the Cartesian plane as the infinite plane formed by all ordered pairs of real numbers
3. Explanation of the relation between the x-value and the y-value of a point on the graph of the linear equation $ax + by = c$, $a \neq 0$ and $b \neq 0$
4. Graphs of simple formulas, as
   $d = rt$, where r or t has a definite value
   $A = ab$, where a or b has a definite value

Suggested Time

Three weeks

Objectives

To learn how to solve problems by graphing on the Cartesian plane

To emphasize the correlation between algebra and geometry through new experiences

Teaching Suggestions

In the outline the words "Cartesian plane," "ordered pair," "relation," and "function" are used. The more traditional names, which are in the basic textbooks, may be used. This unit completes the course in Algebra I. If time allows, a general review, drill, and solution of all types of word problems may be given.

Related Activities

The teacher may consider the following activities for the students:

B. Solution of a System of Linear Equations in Two Unknowns by Graphing

C. Solution of a System of Linear Equations in Two Unknowns by the Addition or Subtraction Method

D. Solution of a System of Linear Equations in Two Unknowns by the Substitution Method

E. Solution of Word Problems of Various Types: Number Relation, Coin, Interest, Mixture, Digit, Rate-Time, Distance, and Geometric Problems

use of State Street and Madison Street as coordinate axes to locate prominent buildings, students' homes, parks, and places of recreation

solution of puzzle problems which requires two unknowns and two equations.

References

Bakst. Mathematics - Its Magic and Mastery, pp. 329-358.

Court. Mathematics in Fun and in Earnest.

School Mathematics Study Group. Mathematics for High School: First Course in Algebra, Part III, pp. 405-463.

Basic and auxiliary textbooks

Suggested Time

Three weeks (nineteenth week)
Review and Examination (twentieth week)

ALGEBRA II

VIII. SPECIAL PRODUCTS AND FACTORING

A. Product of a Polynomial by a Monomial by Inspection

B. Product of a Binomial by a Binomial by Inspection

Objective

To learn the skills of special products and factoring

Teaching Suggestions

Review prime numbers and prime factors. Factoring may be taught by reversing the order of the distributive property. Since 3 (x + y) = 3x + 3y is an

C. Factoring of Polynomials

1. Common monomial factors
2. Trinomials by trial

   a. $x^2 + bx + c$

   b. $x^2 + 2xy + y^2$

   c. $x^2 - y^2$ (Considered as a special case of a trinomial with zero the coefficient of the middle term.)

   d. $ax^2 + bx + c$

D. Solution of Simple Quadratic Equations by Factoring

E. Solution of Word Problems Involving Quadratic Equations by Factoring

identity, then $3x + 3y = 3(x + y)$. Practice with numbers first will give an understanding of the principle of factoring. These skills shorten the time and make easier the solution of problems in quadratics.

Related Activities

Have students report on:
prime numbers
the special properties of the prime number 37
other interesting prime numbers
methods used in magic squares and making magic squares
special products in arithmetic, as:
$39 \times 41 = (40 - 1)(40 + 1) = 1600 - 1 = 1599$
$94 \times 86 = (90 + 4)(90 - 4) = 8100 - 16 = 8084$.

View film A Plus B Squared.

References

Kasner and Newman. Mathematics and the Imagination, pp 64, 187-188.

Newman. The World of Mathematics, Vol. I, pp. 498-520.

School Mathematics Study Group. Mathematics for High School: First Course in Algebra, Part II, pp 247-282.

Basic and auxiliary textbooks

Suggested Time

Three weeks

IX. FRACTIONS

A. Reduction of Fractions to Lowest Terms

1. Number fractions
2. Algebraic fractions

B. Multiplication and Division of Fractions

C. Addition and Subtraction of Fractions

D. Simplifying of Complex Fractions

E. Simplifying of Polynomials

1. Reduction of improper fractions to mixed expressions
2. Changing of mixed expressions to improper fractions

Objective

To review the operations with fractions and to use these skills with polynomials

Teaching Suggestions

Perform each operation of fractions first with numbers and then with polynomials. Emphasize the fact that the rules for number fractions learned in arithmetic are the same rules for algebraic fractions.

Related Activities

On bulletin board post algebraic references found in newspapers and magazines.

List common formulas which contain fractions.

References

Bakst. Mathematics - Its Magic and Mastery, pp. 113-124.

Gardner. Mathematical Puzzles of Sam Lloyd.

School Mathematics Study Group. Mathematics for High School: First Course in Algebra, Part II, pp. 232-241.

Basic and auxiliary textbooks

Suggested Time

Three weeks

X. SOLUTION OF FRACTIONAL EQUATIONS

A. Linear Equations

B. Quadratic Equations by Factoring

C. Simple Systems of Linear Simultaneous Equations

D. Solution of Word Problems Involving Fractions

1. Checking of equations by substitution
2. Changing of the subject of formulas

E. Solution of Problems by Using Proportion

1. Ratio
2. Proportion
3. Variation (optional)

Objective

To apply the skills learned in Unit IX to problem solving

Teaching Suggestions

The basic concepts in algebra have now been taught. Apply these concepts to both equations and word type problems. A general review would be helpful. Most textbooks give such a review at the end of the chapter on fractional equations.

Related Activities

Have students:

find, experimentally, the ratio of the circumference of a circle to its diameter, using globe and measuring tape from NDEA materials

apply the principles of ratio and proportion in making dioramas of the classroom, stage, and other school areas

show how ratio is used to determine sizes and speeds of pulleys

present a research report on the historical development of fractions.

References

Basic and auxiliary textbooks

Suggested Time

Four weeks

XI. POWERS, ROOTS, AND RADICALS

A. Finding of Powers and of Roots by Inspection

B. Extraction of Square Roots

1. Algebraic polynomials which are perfect squares
2. Natural numbers which are perfect squares
3. Natural numbers which are not perfect squares (irrational roots)
4. Expression of square roots with the exponent $\frac{1}{2}$

C. Simplifying of Radicals

D. Operations with Radicals

1. Rationalizing of the denominators of fractions
2. Addition and subtraction of radicals
3. Multiplication and division of radicals

E. Solution of Problems Involving Square Root

1. Radical equations
2. Pythagorean theorem
3. Word problems

Objectives

To review and enlarge the concept of irrational numbers and deepen understanding of real numbers

To develop skill in operations with powers, roots, and radicals

Teaching Suggestions

Review the meaning of rational numbers and irrational numbers. Emphasize the fact that roots occur more often as irrational numbers than as rational numbers.

Related Activities

Have students report on:
development of the radical symbol
applications of the Pythagorean theorem in finding inaccessible distances
the life and work of Pythagoras.

View film Irrational Numbers.

References

Hooper. The River Mathematics, pp. 131-134, 167-179.

Newman. The World of Mathematics, Vol. I, pp. 79-89.

Basic and auxiliary textbooks

Suggested Time

Four weeks

XII. QUADRATIC EQUATIONS

A. Review of the Solutions of Quadratic Equations by Factoring

1. Incomplete quadratic equations
2. Word problems

B. Solution of Quadratic Equations by Completing the Square

C. Derivation of the Quadratic Formula

1. Solution of quadratic equations by formula
2. Determining of the nature of the roots by using the discriminant, $b^2 - 4ac$
3. Finding of the sum and products of the roots of a quadratic equation by use of the quadratic formula (optional)

$ax^2 + bx + c = 0$

$r_1 + r_2 = -\frac{b}{a}$

$r_1 r_2 = \frac{c}{a}$

D. Solution of Word Problems Involving Quadratic Equations

E. Graphing of Quadratics

1. Parabola

$y = ax^2 + bx + c$

Objective

To use the skills learned to solve quadratic equations

Teaching Suggestions

The relation between the degree of an equation and number of its roots is important. Have the students discover as much as possible about the roots of an equation before solving it. Some concepts of the theory of equations are included to prepare the student for more advanced courses in mathematics.

Related Activities

Have students report on:

programing for IBM and other kinds of business machines

the part equations have played in the release of atomic energy

the Thomas Harriot (1560-1621) method of solving quadratic equations by factoring

the opinions of Peacock of England and René Descartes on negative roots.

References

Logsdon. A Mathematician Explains, pp. 49-57.

School Mathematics Study Group. Mathematics for High School: First Course in Algebra, Part III, pp. 505-510.

Weeks and Adkins. First Course in Algebra, pp. 367-386.

Basic and auxiliary textbooks

2. Circle (optional)

$x^2 + y^2 = r^2$

3. Ellipse (optional)

$b^2x^2 + a^2y^2 = a^2b^2$ or $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

4. Hyperbola (optional)

$b^2x^2 - a^2y^2 = a^2b^2$ or $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Suggested Time

Five weeks (nineteenth week)
Review and examination (twentieth week)

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PLANE GEOMETRY

The regular high school geometry course is planned for college-capable students. The students more talented in mathematics should take the honors mathematics course.

The regular course in geometry is basically a traditional Euclidean approach with less emphasis on the number of required theorems. Whenever possible, two-dimensional concepts are extended to three dimensions.

If students have been introduced to set theory in algebra, this unifying idea can be extended and stressed throughout geometry.

Provision for individual differences and enrichment, according to the abilities of students in each class of regular geometry, can be varied through referring to the Teaching Suggestions and Related Activities.

The concepts of plane geometry are developed with many practical applications at the same time that the students are advancing in systematic logical thought.

PLANE GEOMETRY

I. INTRODUCTION TO PLANE GEOMETRY

A. Use of Measurement and Construction Instruments (Review)

1. Linear units
2. Measure of angles
3. Compass

B. Constructions (Review)

1. Bisectors
2. Perpendiculars
3. Angles equal to given angles
4. Notation

C. Definitions

1. Realigment of familiar definitions
2. Mastery of new definitions

D. Axioms and Postulates

E. Simple Proofs

1. Meaning of theorem
2. Examination of hypotheses and conclusions
3. Introductory theorems
   a. Supplements of equal angles are equal
   b. Complements of equal angles are equal
   c. Vertical angles are equal

Objectives

To review the intuitive and informal geometrical concepts previously studied

To recognize inconsistent arguments through careful examination of statements

Teaching Suggestions

Correlate the work of this unit with everyday experiences.

Review, if necessary, the informal geometrical concepts previously studied.

Related Activities

Assign reports on:
the contributions of Euclid and Plato
slogans of manuracturers

References

Bentley and Potts. Geometry. Part One: Discovery by Drawing and Measurement.

Graham. Ingenious Mathematical Problems and Methods.

Johnson. Paper Folding for the Mathematics Class.

Yates. Geometric Tools: A Mathematical Sketch and Model Book.

Basic and auxiliary textbooks

Suggested Time

Three weeks

F. Formal Proofs

1. Inductive approach
2. Deductive approach

II. CONGRUENT TRIANGLES

Objective

To learn the meaning of congruence and its applications

A. Minimum Requirements of Congruence

1. Postulates
   a. SAS = SAS
   b. ASA = ASA
   c. SSS = SSS
2. Examination of corresponding parts

B. Recognition of Congruent Triangles

C. Applications of Congruence

1. Nature
2. Architecture
3. Art
4. Interior decoration
5. Home economics
6. Surveying
7. Musical arrangements

Teaching Suggestions

Keep this unit in the realm of everyday activities. Correlate it with whatever subjects the students are studying. Emphasize that some proofs must be in paragraph form.

Related Activities

Have students construct triangles with sides of 3 inches and 5 inches and included angle of 35°. Let them compare these triangles. Use the same procedure to demonstrate the ASA and SSS postulates.

Have students note the use of congruence in art, in patterns (home economics), in furniture, musical patterns, and the various mechanical units in shops.

View films Angles, Lines and Angles, and Congruent Figures.

References

Ivins. Art and Geometry.

Basic and auxiliary textbooks

Suggested Time

Three weeks

III. PARALLEL AND PERPENDICULAR LINES

A. Conditions for Parallel Lines

1. Classification of angles formed by lines cut by a transversal
2. Introduction to indirect proof

B. Angles Whose Sides are Respectively Parallel or Perpendicular

C. Applications

D. Construction of Parallel Lines

1. Various methods
2. Proof of constructions

E. Extension to Space

1. Diagrams
2. Parallel lines and planes
3. Perpendicular lines and planes
4. Projections

Objectives

To learn the requirements for parallel lines and planes, and perpendicular lines and planes

To understand the indirect method of proof

Teaching Suggestions

In order to develop the concepts of parallel lines and perpendicular lines, use parallel devices and other demonstrative materials obtained through NDEA and the Chicago Board of Education.

Related Activities

For projects, ask students to make models of parallel lines cut by a transversal. Then demonstrate the various kinds of angles involved.

View film Parallel Lines.

References

Hornung. Handbook of Designs and Devices.

Bonola, et al. Non-Euclidean Geometry.

Vergara. Mathematics in Everyday Things

Basic and auxiliary textbooks

Suggested Time

Three weeks

IV. THE ANGLES OF A TRIANGLE

A. Sum of the Interior Angles

1. Relation of this sum to the parallel postulate
2. Sum of the interior and sum of the exterior angles of polygons

B. Other Relations

1. $30^\circ - 60^\circ$ right triangle
2. Scalene triangle
3. Isosceles triangle

C. Extension of Congruence to Right Triangles with Applications

Objectives

To become familiar with the angle relations of triangles

To learn how to prove right triangles congruent

To show that the parallel postulate of Euclidean geometry is basic to the theorems that follow in this course

Teaching Suggestions

Use such visual aids as the geometry demonstration instruments and the chalkboard triangle.

Related Activities

Assign projects that involve making models of the theorems that have been studied.

Demonstrate the use of the $30^\circ - 60^\circ$ right triangle.

Have the students report on the "Theory of Parallels" by Lobachevski, and by Riemann.

References

Freeman and Freeman. Fun with Figures.

Shuster and Bedford. Field Work in Mathematics.

Basic and auxiliary textbooks

Suggested Time

Three weeks

V. QUADRILATERALS

Objectives

To learn the different kinds of quadrilaterals and the special properties of parallelograms and trapezoids

To compare the uses of the quadrilateral with those of the triangle

A. Classification

1. Characterization by number of parallel sides
2. Characterization by equality of sides and kinds of angles

B. Examination of the Properties of Parallelograms

1. Sides
2. Angles
3. Diagonals
4. Altitudes

C. Investigation of the Properties of a Trapezoid

1. Median
2. Base angles
3. Diagonals

Teaching Suggestions

Develop this unit on quadrilaterals by contrasting the rigidity of the triangle with the flexibility of the quadrilateral.

Related Activities

Have students report on:
the use of parallelograms in construction
the principles of parallelograms used in instruments for navigation
the parallelogram law of forces
vectors and vector analysis.

Have students make models of parallelograms with movable parts to demonstrate the theorems studied.

Arrange a trip to a large department store to study the window-display designs. This is a production that will fascinate students.

References

Kellaway. Map Projections.

Weyl. Symmetry.

Basic and auxiliary textbooks

Suggested Time

Two weeks

VI. APPLICATIONS OF PARALLEL LINES

A. Transversals

1. Segments intercepted by parallels
2. Division of a line segment into equal parts

B. Line Segment Joining the Midpoints of Two Sides of a Triangle

1. Relation to base
2. Applications

C. Right Triangle Relationships

1. The midpoint of the hypotenuse
2. The 30° - 60° right triangle relationship (Extension and review)

Objectives

To extend understanding of the properties of parallel lines and their applications

To learn important right triangle relations

To demonstrate the aesthetic quality of geometric figures as well as their uses

Teaching Suggestions

Use the demonstration geometry instruments provided through NDEA to develop the concepts of this unit.

Show students how to divide a line segment into equal parts by the parallel line method.

Related Activities

Have students construct symmetrical figures.

D. Simple Solids

1. Cubes
2. Rectangular solids
3. Pyramids

E. Symmetry

1. Point
2. Line
3. Plane

Find drawings and photographs to illustrate axial symmetry in nature, in architecture, and in engineering designs.

Correlate with biology by asking for specimens of leaves, insects, or moths.

View film Discovering Solids: Volumes of Cubes, Prisms, Cylinders.

References

Brandes. Geometry Can Be Fun.

Ghyka. The Geometry of Art and Life.

Hambridge. Elements of Dynamic Symmetry.

National Council of Teachers of Mathematics. Multisensory Aids in the Teaching of Mathematics. Eighteenth Yearbook.

Ravielli. An Adventure in Geometry.

Basic and auxiliary textbooks

Suggested Time

Two weeks

VII. INEQUALITIES

A. Introduction

1. Review of basic properties and symbols
2. Application to geometry

B. Geometric Relations

1. Triangle with two unequal sides or two unequal angles
2. Triangles with two sides equal respectively and with unequal included angles
3. Triangles with two sides equal respectively and with unequal third sides

C. Applications

1. Other subjects
2. Home situations

Objective

To focus attention on the geometric meaning of order and its application in triangles

Teaching Suggestions

Since most of the axioms of inequality are introduced in algebra, the emphasis here should be on inequality relationships in triangles.

Related Activities

Have students note inequalities in:
- sizes of students
- ages of students
- heights of apartment buildings

Have students prove some of the basic properties of inequalities and report on their applications.

References

Wallin. "An Application of Inequalities," The Mathematics Teacher, (February, 1960), pp. 134-135.

Basic and auxiliary textbooks

Suggested Time

Two weeks

VIII. CIRCLES AND THEIR LINES

A. Relations of Equal and Unequal Central Angles, Arcs, and Chords

1. Definitions
2. Properties
3. Constructions
4. Applications

B. Relations of Diameters Perpendicular to Chords

C. Relations of Tangents to a Circle

D. Relation of Tangent Circles

E. Line of Centers of Intersecting Circles

Objectives

To learn the basic concepts of circles and their lines

To relate the information gained in previous units to the new materials on circles

Teaching Suggestions

Ask students to list objects in the home and school where circles are used.

Use any circular devices available to point out the relationships involved.

Related Activities

Assign special reports on the use of the circle in:
department stores
automobiles
explanation of sets.

References

Pickett. "Length of Chords and Their Distances from the Center," The Mathematics Teacher, Vol. L, No. 5 (May, 1957), pp. 325-326.

Rademacher and Toeplitz. The Enjoyment of Mathematics. pp. 160-163.

Basic and auxiliary textbooks

Suggested Time

Two weeks

Geometry II

IX. MEASUREMENT OF ANGLES AND ARCS

A. Classification of Angles According to their Formation

1. Two radii
2. Two chords
3. Tangent and chord
4. Two secants
5. Two tangents
6. Tangent and secant

B. Parallel Lines Intersecting a Circle

C. Constructions

D. Applications

1. Lenses
2. Gear wheels

Objective

To learn the measurement of various angles formed by lines of a circle

Teaching Suggestions

Relate measurement of angles and arcs to practical situations by showing examples in other fields.
Use the circular device provided by NDEA and the Chicago Board of Education to demonstrate the formation of various angles and their relationship to the arcs.

Related Activities

Have students find examples of circles and angles of circles in art and in architecture.

References

Rademacher and Toeplitz. The Enjoyment of Mathematics. pp. 160-163.

Shuster and Bedford. Field Work in Mathematics.

Thompson. On Growth and Form.

Basic and auxiliary textbooks

Suggested Time

Two weeks

X. LOCI

A. Definition

1. Fundamental terms
2. Relation to other topics

B. Fundamental Relationships

1. Intuitive discovery
2. Formal proof

C. Compound or Intersecting Loci

D. Applications

1. School
2. Home
3. Literature
4. Navigation

E. Three Dimensions

Objective

To review the basic concepts of loci and extend them to include logical proof

Teaching Suggestions

Emphasize that a locus is a set of points. This is an appropriate time to review and extend the set concept and to coordinate it with equations in algebra.

Related Activities

Demonstrate the use of loci in the classroom.

Have students report on the conic sections.

Recall incidents in literature where treasure hunts have used the locus concept. Have students plan trips within a certain locus.

View film Locus.

References

Hoffman and Osborn. "Concerning Simultaneous Solutions of Polar Equations by the Graphical Method," The Mathematics Teacher, Vol. LIII, No. 2.

National Council of Teachers of Mathematics. Multi-Sensory Aids in Mathematics. Eighteenth Yearbook.

Basic and auxiliary textbooks

Suggested Time

Two weeks

XI. CONCURRENT LINES AND TRIANGLES

A. Definitions

1. Circumcenter
2. Incenter
3. Orthocenter
4. Centroid

B. Intersections of Loci

Objective

To learn the definitions and theorems involved in the circumscribing and inscribing of circles

Teaching Suggestions

Use the dynamic geometry instruments or pegboard provided through NDEA to demonstrate these relationships.

Use the physical characteristic of the centroid to illustrate the center of gravity.

Related Activities

Have students make cardboard models of scalene triangles, using rubber bands as the perpendicular bisectors of the sides, as the bisectors of the angles, and as the medians.

References

Koral. "A New Look at Medians," The Mathematics Teacher, Vol. LI, No. 2 (February, 1958), p. 123.

National Council of Teachers of Mathematics. Multi-Sensory Aids in Mathematics. Eighteenth Yearbook.

Basic and auxiliary textbooks

Suggested Time

Two weeks

XII. RATIO AND PROPORTION

A. Definitions

B. Fundamental Properties of Proportion

C. Proportional Line Segments and Triangles

1. Line parallel to one side of a triangle and intersecting the other two, and the converse
2. Bisector of an angle of a triangle
3. Bisector of an exterior angle of a triangle

D. Extension to Construction

1. Proportional parts of transversals cut off by parallel lines
2. Fourth proportional
3. Internal and external division of line segments

E. Parallel Planes Intersecting Lines

Objective

To review ratio and proportion and extend the concept to geometry

Teaching Suggestions

Demonstrate prortional parts by use of various devices. Assign projects related to division of line segments.

Related Activities

Have pupils report on:
use of ratio and proportion in art and architecture
application of ratio and proportion in calculating sizes and speeds of pulleys and gears
construction of slide rule.

References

Gardner. Instruments for the Enrichment of Secondary School Mathematics.

Thompson. On Growth and Form.

Basic and auxiliary textbooks

Suggested Time

Three weeks

XIII. SIMILAR POLYGONS

A. Fundamental Concepts

1. Proof of similarity of triangles
2. Similar polygons
3. Ratio of perimeters of similar polygons

B. Constructions

1. Triangles similar to a given triangle
2. Polygons similar to given polygons

C. Proportional Lines of Circles

D. Practical Applications

1. Scale drawings
2. Surveying
3. Photographs

Objectives

To learn the conditions that are necessary for similar triangles and for similar polygons

To realize the importance of similar figures in solving practical problems

Teaching Suggestions

By means of suitable figures or models, demonstrate the difference between equality, congruence, and similarity. Emphasize the many practical applications of the unit.

Related Activities

The exploratory approach may be used here. Use the pantagraph and proportional dividers provided through NDEA to enlarge and reduce designs and maps. Assign projects such as making models and designs.

View film Similar Triangles.

References

Shuster and Bedford. Field Work in Mathematics.

Basic and auxiliary textbooks

Suggested Time

Three weeks

XIV. PYTHAGOREAN RELATIONSHIPS

A. Right Triangle Relationships

1. Altitude to the hypotenuse
2. Pythagorean theorem and its converse
3. Projections

B. Constructions

1. Mean proportional
2. Golden rectangles

C. Applications

D. Extension to Trigonometry

Objective

To learn the Pythagorean theorem, at least one of its proofs, its converse, and some of its many applications

Teaching Suggestions

Use the pocket transit, simplified transit, level, and sextant provided through NDEA to solve practical problems in this unit. Emphasize pythagorean converse.

Integrate arithmetic, algebra, geometry, and trigonometry here.

Related Activities

Have students report on:
contribution of Thales to mathematics
life of Pythagoras
Garfield proof of the Pythagorean theorem
other proofs of the theorem

References

Johnson and Glenn. The Pythagorean Theorem.

Ransom. Geometry Growing.

Basic and auxiliary textbooks

Suggested Time

Three weeks

XV. AREAS OF POLYGONS

A. Proofs of Formulas

1. Rectangle
2. Parallelogram
3. Triangle
4. Trapezoid

B. Ratios of Areas

1. Parallelograms
2. Triangles

C. Area Relations

1. Similar triangles
2. Other similar polygons

D. Hero's Formula

E. Constructions

1. Polygon equivalent to a triangle
2. Parallelogram equivalent to a square

Objectives

To review areas of surfaces and learn how to construct equivalent figures

To learn how to prove the various area formulas

Teaching Suggestions

The work of this unit should continue the integration of the areas of mathematics. The proofs of the theorems use algebra and the solutions of the exercises involve ratio and proportion. To do this teaches correct selection of formulas and proper substitution of given data.

Related Activities

Have students find the area of an irregular plot of ground that can be divided into the figures studied in this unit.

View film Areas.

References

James (ed.). The Tree of Mathematics, pp. 41-56.

Basic and auxiliary textbooks

Suggested Time

Two weeks

XVI. REGULAR POLYGONS AND THE CIRCLE

A. Definitions

B. Constructions

1. Circle circumscribed about a regular polygon
2. Circle inscribed in a regular polygon
3. Inscribing and cirsumscribing polygons

C. Area of a Polygon

D. Similar Polygons

1. Similarity of regular polygons with same number of sides
2. Perimeters and areas of regular polygons with same number of sides

E. Applications

Objectives

To learn the constructions involving regular polygons and their proofs

To develop the formula for the area of a regular polygon

Teaching Suggestions

This unit should teach the relationships among regular inscribed and circumscribed polygons and circles as well as the formulas for the perimeters and areas of these polygons. Use the faces of the regular polygons provided through NDEA.

Related Activities

Assign projects pertaining to industrial design and its relation to regular polygons.

Furnish as many practical examples as possible before generalizing.

Problems concerning the furnishing of homes will be appropriate here.

References

Hornung. Handbook of Designs and Devices.

Morris. Geometrical Drawing for Art Students.

Basic and auxiliary textbooks

Suggested Time

One week

XVII. CIRCLE MEASUREMENT

A. Derivation of Formulas

1. Circumference
2. Area

B. Determination of Ratios

1. Areas of two circles
2. Value of pi

C. Applications

1. Areas of sectors and segments
2. Relationships of various parts

D. Extension to Three Dimensions (Optional)

Objective

To learn the derivation of formulas for circumference and area of circle and applications of these

Teaching Suggestions

Review the intuitive development of the circumference and area formulas. Relate the perimeter of a many-sided regular polygon to the circumference of a circle.

Related Activities

Have students report on:
Archimedes' method of circle measurement
use of circles in architecture and technology.

References

Ghyka. The Geometry of Art and Life.

Ravielli. An Adventure in Geometry.

Basic and auxiliary textbooks

Suggested Time

Two weeks

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ALGEBRA III and IV

This course is intended primarily for those students who desire to continue their mathematical education in high school beyond the tenth grade, but are not necessarily planning to major in mathematics or science in college. Anticipating the Chicago plan of annual promotion, the outline is given for a one-year course. However, the content is divided into two semesters, so that students may elect only the first semester, or postpone the second semester for a later time.

In most Chicago public high schools, Algebra III follows plane geometry. Thus, a year will have elapsed for most students since they were last enrolled in an algebra class. Therefore, basic skills taught in elementary algebra should be reviewed. Verbal problems can be used to make this review more interesting. This also will give practice in translating the English sentence into the algebraic sentence.

Whenever feasible, the content of this course should be presented in the spirit of modern mathematics, with the maturity level of the learner constantly in mind. Specifically, it is recommended that the development of the number system and the unit on sets be taught at the beginning of Algebra III. These units are expanded in the Appendix. The units on logarithms and the slide rule, numerical trigonometry, and statistics should prove useful as a background for the natural and behavioral sciences.

Some flexibility in covering the content of this course should be allowed, since the ability of the students will vary considerably from class to class. The outline presented here is planned for students of average ability. Optional units may be omitted at the discretion of the teacher.

The potential value of this course is considerable. It not only will provide the student with useful mathematical tools, but will expose him to vistas of mathematical thought which will help him to understand and appreciate the world of science and technology.

ALGEBRA III

I. INTRODUCTION

A. Development of The Number System Through the Real Numbers (Review)

B. Extension of the Number System to Include Complex Numbers

C. Theory of Sets (Review)

1. Basic concept of sets
2. Equal and equivalent sets
3. Subsets
4. Operations with sets
   a. Diagrams
   b. Intersection and union of sets
   c. Complementary sets

Objectives

To review the historical background of the number system

To extend the number system to include complex numbers

To review the modern concept of sets

Teaching Suggestions

The development of the number system is expanded in the Appendix of this book. This material should be reviewed, and the history of the extension of the number system to include complex numbers should be given. The expansion of the theory of sets, in the Appendix, should be reviewed and the vocabulary emphasized. Some of the exercises in the Appendix and in the references should be worked.

Related Activities

Have students make reports and charts on the following:
Babylonian numerals
contributions of ancient Babylonians to mathematics
Arabic numerals
binary number system
number system with base twelve
set theory
history of sets in mathematics.

View film Sentences and Solution Sets.

References

Aiken and Beseman. Modern Mathematics: Topics and Problems.

Courant and Robbins. What is Mathematics?

Johnson and Glenn. Sets, Sentences, and Operations.

Krickenberger and Pearson. An Introduction to Sets and the Structure of Algebra.

Logsdon. A Mathematician Explains.

Woodward and McLennan. Elementary Concepts of Sets.

Young. Digital Computers and Related Mathematics.

Suggested Time

Two weeks

II. FUNDAMENTAL OPERATIONS (Review)

A. Signed Numbers
   1. Monomials
   2. Polynomials

B. Order of Operations

C. Removal and Insertion of Parentheses

D. Commutative, Associative, and Distributive Properties

E. Solution of Equations by Above

Objective

To maintain the skills taught in elementary algebra with emphasis on the commutative, associative, and distributive properties and inequalities

Teaching Suggestions

Since a year has elapsed when most of the students studied algebra, the basic skills should be reviewed. The commutative, associative, and distributive properties should be emphasized. More work on inequalities should be given. Solution of word problems involving these basic skills should be included.

Properties

1. Linear equations
2. Word problems
3. Inequalities

Related Activities

Suggest that students, at the eleventh grade level, interested in mathematics, participate in the activities of a mathematics club.

References

Basic and auxiliary textbooks

Suggested Time

Two weeks

III. SPECIAL PRODUCTS AND FACTORING

Objective

To develop skills in special products and factoring to meet the present needs of the students

A. Special Products (Review)

1. $(a + b)(a + c)$
2. $(a + b)^2$
3. $(a + b)(a - b)$
4. $(a + b + c)(a + b - c) =$

   $\left[(a + b) + c\right] \left[(a + b) - c\right]$
5. $(ax + b)(cx + d)$
6. $(a \pm b)^3$

B. Factoring

Teaching Suggestions

Factoring should be mastered by each pupil in the class to prepare him for the applications of this skill to fractions, to equations, and to problem solving. The main problem is the recognition of patterns, and this requires practice and explicit description of the patterns. Interest can be sustained by presenting different types of factoring.

Related Activities

Have students make reports on the following:

1. Common monomial factor
2. Grouping of terms

   a. $a(x + y) + b(x + y)$

   b. $ax - ay - bx + by$

3. Perfect squares

   a. $x^2 + 2xy + y^2$

   b. $mx^2 + 2mxy + my^2$

4. Difference of two squares

   a. $x^2 - y^2$

   b. $ax^2 - ay^2$

   c. $x^2 + 2xy + y^2 - a^2$

5. Quadratic trinomials

   a. $x^2 + bx + c$

   b. $ax^2 + abx + ac$

   c. $mx^2 + nx + d$

6. Sum or difference of two cubes

   a. $x^3 + y^3$

   b. $ax^3 + ay^3$

   c. $x^3 - y^3$

   d. $ax^3 - by^3$

interesting number problems based on the product of (a + b) (a - b) and other special products
prime numbers
Eratosthenes "sieve"
special properties of certain prime numbers.

References

National Council of Teachers of Mathematics. The Growth of Mathematical Ideas, K-12, pp. 397-402.

Newman. The World of Mathematics, Vol. 1, pp. 500-508.

Basic and auxiliary textbooks

Suggested Time

Two weeks

7. Solution of equations by factoring
8. Solution of word problems by factoring

IV. FRACTIONS

A. Fundamental Principles (Review)

1. Reduction of fractions to lowest terms
2. Multiplication and division of fractions
3. Changing of fractions to equivalent fractions
   a. By multiplying both numerator and denominator by the same number (not zero)
   b. By dividing both numerator and denominator by the same number (not zero)
4. Addition and subtraction of fractions
5. Changing of fractions to equivalent fractions by change of signs
6. Changing of mixed expressions to improper fractions
7. Changing of improper fractions to mixed expressions
8. Simplification of complex fractions

B. Solution of Linear Equations with Fractional Coefficients

Objectives

To maintain the skills of operations with fractions

To apply these skills to problem solving

Teaching Suggestions

The students have had considerable practice with arithmetic fractions and algebraic fractions. However, when a familiar idea is presented in an unfamiliar context, the idea, usually, must be retaught.

Related Activities

Have students make reports on application of fractions to the sciences.

Have students make reports on various types of tolerance in manufacturing.

References

Basic and auxiliary textbooks

Suggested Time

Two weeks

C. Solution of Various Types of Word Problems Involving Fractions

V. RELATIONS

A. Variables and Constants

B. One-to-One Correspondence

C. Functions

1. Tables
2. Formulas
3. Graphs

D. Graphing of Algebraic Functions

E. Ratio and Proportion (Review)

F. Direct and Inverse Variation

1. Solution of formulas
2. Solution of word problems

Objectives

To develop a clear understanding of the concept of relation and function

Teaching Suggestions

If some modern mathematics has been introduced in this course, relations, as sets of ordered pairs, could be introduced. Functions, as a special kind of relation, should be explained.

The concept of one-to-one correspondence is important and should be reviewed.

Related Activities

Have students make reports on the Following:
the history and various meanings of the word "function" in mathematics
the graphic solution of parallel resistarce
the use of graphs to show relations in business, industry, science, and research.

Have students collect examples of direct and inverse variations.

View film Concept of Function.

VI. LINEAR SYSTEMS

A. Solution of Linear Systems of Equations in Two Variables by Graphing

1. Explanation of Cartesian plane
2. Explanation of ordered pair
3. Explanation of domain and range
4. Independent, dependent, and inconsistent systems of equations

B. Solution of Systems of Equations in Two Variables by Addition and Subtraction

C. Solution of Systems of Linear Equations in Two Variables by Substitution

D. Solution of Word Problems by Using Two Unknowns

References

Griswold, Keedy, and Schact. Contemporary Algebra and Trigonometry, pp. 77-105.

Basic and auxiliary textbooks

Suggested Time

Two weeks

Objectives

To review the solution of linear systems of equations in two variables

To expand the skills of solution to systems of linear equations in three variables

To learn how to evaluate determinants

Teaching Suggestions

The review of solution of linear equations in two unknowns by algebra and by graphs should be as rapid as possible for a particular class. Inequalities should be included in the review. The third order of determinants should only be taught after mastery of the second order. Determinants of the third order may be omitted if time does not allow.

Related Activities

Have students report on the following:

E. Solution of Systems of Linear Equations in Three Unknowns by Algebraic Methods

F. Evaluation of Determinants of the Second Order and the Third Order

G. Solution of Systems of Linear Equations by Determinants of the Second Order and of the Third Order

H. Solution of Word Problems by Using Three Unknowns

practical problems which require two equations in two variables for solution

the mathematical contributions of Rene Descartes.

References

Bakst. Mathematical Puzzles and Pastimes, pp. 84-96.

Griswold, Keedy, and Schact. Contemporary Algebra and Trigonometry, pp. 84-96, 177-186.

School Mathematics Study Group. Mathematics for High Schools: First Course in Algebra.

Basic and auxiliary textbooks

Suggested Time

Two weeks

VII. SQUARE ROOT, EXPONENTS, RADICALS, AND COMPLEX NUMBERS

A. Square Root of a Number

1. Arithmetical method
2. Use of a table

   a. Perfect squares
   b. Numbers, not perfect squares, by interpolations

B. Exponents

Objectives

To maintain by review and advanced work the skills of square root, exponents, and radicals

To learn the concepts of complex numbers and to acquire skills of operations with them

Teaching Suggestions

Complex numbers were explained at the beginning of this course in the development of the number system. The form, a + bi, as a pure imaginary number when a = 0 should be discussed. The idea that pure imaginary

1. Meaning of positive and negative exponents
2. Meaning of fractional and zero exponents
3. Properties of exponents
4. Solution of exercises involving the properties of exponents

C. Radicals

1. Extraction of the indicated root of an integer and of a fraction (rationalization)
2. Addition and subtraction
3. Multiplication and division
4. Solution of radical equations
5. Solution of word problems involving square root and radicals

D. Complex Numbers

1. Understanding of the meaning of a + bi
   a. a + bi, a real number when b = 0 (real numbers, a subset of complex numbers)
   b. a + bi, a pure imaginary number when a = 0 (pure imaginary numbers, a subset of complex numbers)
2. Simplifying of pure imaginary numbers

$$\sqrt{-a} = i\sqrt{a} \quad (a > 0)$$

numbers form a subset of the complex number system should be considered before the operations with imaginary numbers.

Related Activities

Have students report on:
graphs of complex numbers
Gaussian complex integers.

View film Irrational Numbers.

References

Commission on Mathematics. Report of the Commission on Mathematics: Appendices, pp. 58-63.

Courant and Robbins. What is Mathematics?

Hooper. The River Mathematics, pp. 381-389.

Kasner and Newman. Mathematics and the Imagination, pp. 93-103.

Newman. The World of Mathematics, Vol. 1

Basic and auxiliary textbooks

Suggested Time

Two weeks

3. The four fundamental operations with pure imaginary numbers and complex numbers
4. The complex plane
   a. Representation of complex numbers on the complex plane
   b. Addition and subtraction of complex numbers by graphing

VIII. QUADRATIC EQUATIONS IN ONE VARIABLE

A. Review of Solutions of Quadratic Equations

1. Factoring
2. Completing the square
3. Quadratic formula
4. Graphing (parabola)

B. Review

1. Nature of the roots by using the discriminant
2. Sum and product of the roots
3. Relation between the roots and coefficients

C. Forming of a Quadratic Equation from its Roots

D. Solving of Quadratic Equations with Complex Roots

Objectives

To apply the skills mastered to the solution of quadratic equations

To study some elementary concepts of the theory of equations

Teaching Suggestions

Students should realize that all the skills studied are used in the solution of equations.

This unit provides a good review.

The use of the discriminant is important in the theory of equations.

E. Solving of Radical Equations Involving Quadratics

F. Solving of Word Problems Involving Quadratic Equations

Related Activities

Have students report on:
the many uses of the parabola in everyday life
the part equations played in the development of radio and television
the solution of quadratic equations by Al-Khowarizmi of Arabia, Sridhara of India, and Omar Khayyam of Persia.

References

Allendoerfer and Oakley. Fundamentals of Freshman Mathematics.

May. Elements of Modern Mathematics.

Basic and auxiliary textbooks.

Suggested Time

Two weeks

IX. QUADRATIC EQUATIONS IN TWO VARIABLES

Objectives

To review the graph of a parabola

To apply the method of graphing to the circle, the ellipse, and the hyperbola

To apply the knowledge of graphing to solving systems of two equations in two variables

A. Explanation of the Conic Sections

1. Geometric demonstration
2. Algebraic representation of the conic sections
   a. The parabola and its equations

(1) Maximum or minimum value
(2) Solution by graphing of a quadratic equation of the form $ax^2 + bx + c = 0$ when roots are real

b. The circle and its equation of the form

$$x^2 + y^2 = r^2$$

c. The ellipse and its equation of the form

$$b^2x^2 + a^2y^2 = a^2b^2$$

or $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

d. The hyperbola and its equations of the forms

$$b^2x^2 - a^2y^2 = a^2b^2$$

or $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, and $xy = k$

e. Analytic properties of the conic sections (optional)

B. Simultaneous Quadratic Equations in Two Variables

1. Graphic and algebraic solutions with one quadratic equation and one linear equation
2. Solutions with both equations of the quadratic form (optional)

Teaching Suggestions

The models of the intersection of planes with a conical surface or cone should be used to demonstrate the four principal conic sections. The general algebraic equation of the conic section demonstrating each geometric figure should be given on the chalkboard. Several models of the N.D.E.A. materials may be used for this demonstration. Also, each conic section may be generated by a moving point. Give the locus problem for each.

Related Activities

With the class discuss the following:
the orbits of the stars and planets
the practical applications of the conic sections.

Have students make reports on the solution of systems of two quadratic equations by various algebraic methods.

References

Commission on Mathematics. Report of the Commission on Mathematics: Appendices, pp. 47-57.

Hart. College Algebra (4th ed.), pp. 157-173.

Mallory and Fehr. Senior Mathematics for High Schools pp. 234-268, 407-410.

Rosenbach, Whitman, Meserve, and Whitman. College Algebra (4th ed.), pp. 202-216.

Basic and auxiliary textbooks

Suggested Time

Three weeks; review and tests, one week

ALGEBRA IV REGULAR

X. DETERMINANTS AND LINEAR SYSTEMS

Objectives

To understand the concepts of matrix and determinant

To learn how determinants can be used in solving systems of equations (Cramer's rule)

To relate the algebraic solutions of linear equations in three unknowns with the geometry of planes and their intersections

A. Matrices

B. Second Order Determinants (Review)

1. **Evaluation of second order determinants**
2. **Solution of linear equations in two unknowns by determinants**

C. Determinants of the Third Order

1. **Expansion by augmenting the determinant**
2. **Expansion by minors**
3. **Solution of three linear equations in three unknowns by determinants (Cramer's rule)**
4. **Consistent, inconsistent, and dependent systems; interpretations in terms of planes and their intersections**
5. **Problems leading to simultaneous systems**

D. Determinants of Higher Order (Optional)

Teaching Suggestions

Determinants form only a part of the larger subject of matrix theory. Essentially, a matrix is a rectangular array of numbers. With each square matrix there is associated a number called the determinant of the matrix. In this unit, determinants arise in connection with the solution of systems of linear equations. Cramer's rule should be derived by direct elimination. Inconsistent and dependent systems should be interpreted in terms of parallel lines and planes.

Related Activities

Have students report on the following:

1. The "double subscript" notation
2. General rule for expansion
3. Important properties
4. Reduction to canonical forms

systems of equation in which the number of equations is not equal to the number of unknowns

homogeneous equations

Cramer's rule derived by using cofactors.

References

Basic and auxiliary textbooks

Suggested Time

Two weeks

XI. LOGARITHMS AND THE SLIDE RULE

A. Preliminary Topics

1. Review of the properties of exponents
2. The logarithmic concept through powers of 2
3. Definition of a logarithm
4. Conversion from exponential to logarithmic form and vice versa
5. Exponential and logarithmic equations whose solutions can be found by inspection
6. Fundamental properties of logarithms

B. Common Logarithms

1. Approximate numbers, significant figures and the correct use of approximate numbers

Objectives

To acquire understanding of the meaning of logarithms and their properties

To learn the use of a table of logarithms

To learn how to compute with the aid of logarithms

To solve exponential and logarithmic equations

To use the slide rule in solving problems

Teaching Suggestions

Logarithms are important in mathematics both as a tool and as an integral part of mathematical analysis. In this unit the emphasis is on the tool aspects of logarithms. Nevertheless, the analytical aspect can be introduced through the solutions of exponential and logarithmic equations.

2. Scientific notation
3. The characteristic of a number
4. The mantissa of a number
5. Interpolation
6. Antilogarithms

The construction of tables of integral powers of 2 or 10 could serve as an introduction to the unit.

C. Computations by Logarithms

1. Products, quotients, powers, and roots
2. Exponential equations
3. Logarithmic equations
4. Applications

Related Activities

Have students report on the following:
compound interest formula
system of natural logarithms
conversion from one system of logarithms to another
semilogarithmic coordinates and graphs
history of logarithms and the slide rule.

View films Slide Rule: Multiplication and Division
Slide Rule: Percentage, Proportion, Squares and Square Root.

D. Theory and Practice of the Slide Rule

References

Basic and auxiliary textbooks

Suggested Time

Three weeks

XII. NUMERICAL TRIGONOMETRY

Objectives

A. The Six Trigonometric Functions of an Acute Angle

1. Review of the concepts of ratio and function
2. Definitions of the functions in terms of the ratios of the sides of a right triangle
3. Functions of special angles

To define the six trigonometric functions and derive some of their properties

To learn the use of the tables of natural trigonometric functions and the tables of the logarithms of the functions

To learn to solve triangles, and find unknown distances and angles

4. Tables of the trigonometric functions
5. Inverse use of the tables
6. Variations of the functions
7. Relations among the functions

B. Applications

1. Solution of right triangles
2. Indirect measurement of distances and angles
3. Solution of problems involving vectors

C. Functions of Any Angle (Optional)

1. Definition of the functions in terms of coordinates
2. Identification of the new definitions with the old ones when the angle is acute
3. Reduction formulas and identities
4. The law of sines
5. The law of cosines

D. Applications (Optional)

1. Solution of the four cases of oblique triangles
2. Solution of problems from navigation and surveying
3. Solution of problems involving vectors
4. Derivation of formulas for the areas of triangles and regular polygons

Objectives

To develop some facility in proving simple trigonometric identities and solving simple trigonometric equations

Teaching Suggestions

This unit is an introduction to trigonometry. It emphasizes those concepts and techniques which the student can utilize immediately in other parts of mathematics and in science. The present trend in trigonometry is to emphasize analytical concepts. This can be accomplished by defining the functions in terms of coordinates. Radian measure and trigonometric functions of real numbers are not included in this unit.

Related Activities

Have students report on:
the use of trigonometry in surveying, navigation, physics, and astronomy
solving problems in geometry by trigonometry.

References

Basic and auxiliary textbooks

Suggested Time

Two weeks

XIII. PROGRESSIONS AND THE BINOMIAL THEOREM

A. Arithmetic Progressions

1. Definition
2. Formula for the nth term
3. Formula for the sum of n terms
4. Insertion of arithmetic means
5. Applications

B. Geometric Progressions

1. Definition
2. Formula for the nth term
3. Formula for the sum of n terms
4. Insertion of geometric means
5. Applications

C. Infinite Geometric Progressions

1. The limit concept
2. Formula for the sum of an infinite geometric progression
3. Repeating decimals
4. Applications

D. Binomial Theorem

1. Binomial expansion formula
2. Rule for finding the successive terms
3. The general term of the expansion
4. Expansion of $(1 + x)^r$ where r is a rational number
5. Applications to roots and powers

Objectives

To acquire mastery in finding the general term and the sum of the terms of arithmetic progressions and geometric progressions

To find the sum of an infinite geometric progression, and to convert a repeating decimal to a fraction

To learn the binomial expansion formula and its use in computations

Teaching Suggestions

Beginning with this unit, the student is introduced to "combinational mathematics," that is expressions consisting of many terms which combine in special ways. For the most part, the mathematics is straightforward and not too difficult. Only in the cases of infinite geometric progressions and the binomial series will difficulties arise. An intuitive approach to the limit concept will have to suffice for the present. Student interest in this unit can be augmented with a suitable choice of examples and problems.

Related Activities

Have students report on the following:
- Pascal's triangle and its properties
- factorial numbers
- harmonic progressions
- the multinomial theorem
- the mathematics of annuities.

References

Basic and auxiliary textbooks

XIV. PERMUTATIONS, COMBINATIONS, AND PROBABILITY

Suggested Time

Three weeks

Objectives

To acquire an understanding of the concepts "permutation," "combination," and "probability"

To derive the basic formulas of this unit, and to apply these formulas to a variety of situations

A. Permutations

1. Multiplication principle in counting
2. Definition of a permutation
3. Permutations of n different things taken r at a time ($n \geq r$)
4. Permutations when some of the things are alike
5. Circular and linear permutations
6. Applications

B. Combinations

1. Definition of a combination
2. Combination of n different things taken r at a time ($n \geq r$)
3. Formula for the total number of combinations
4. Proof of the binomial theorem by combination theory (optional)
5. Applications

C. Probability

1. The probability concept

Teaching Suggestions

Permutations and combinations are useful in all fields of mathematics, while probability is the foundation of advanced statistics. Only the most elementary treatment of the topics should be considered. Students will gain insight through problem solving. "Tree" diagrams and set theory may be introduced when feasible. The identification of the combination symbol with the binomial coefficients should be attempted.

Related Activities

Have students report on the following:
results of experiments involving probability
the use of probability in life insurance
permutations and combinations in geometry
the derivation of π by probability
a brief history of probability.

2. Dependent and independent events
3. Mutually exclusive events
4. Mathematical expectation
5. Statistical probability
6. Applications

References

Kemeny, Snell, and Thompson. Introduction to Finite Mathematics, chapter IV.

Basic and auxiliary textbooks

Suggested Time

Three weeks

XV. FUNCTIONS AND EQUATIONS OF HIGHER DEGREE

Objectives

To study the properties of the polynomial function

$$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_n$$

by algebraic and graphical techniques

To find the rational zeros of $f(x)$, and the irrational zeros to any desired degree of accuracy

To explore additional concepts pertaining to the theory of equations

A. The Polynomial Function

1. Standard form of the polynomial function
2. Functional notation
3. Remainder and factor theorems
4. Synthetic division
5. Graph of the polynomial function

B. Real Roots of the Polynomial Equation

1. Location principle
2. Upper and lower bounds for the roots
3. Rational roots
4. Depressed equations
5. Irrational roots

C. Miscellaneous Topics (Optional)

Teaching Suggestions

In his previous study of algebra, the student learned to solve equations of the first and second degrees (or equations reducible to such types). Now he is confronted with the more difficult problem of solving equations of any degree. This

1. Complex roots and surd roots
2. Coefficients in terms of the roots
3. The fundamental theorem of algebra
4. Algebraic solutions of equations
5. Problems leading to equations of higher degree

will require on his part computational skills far beyond his previous experience. Graphical devices and synthetic division will be his main tools.

Related Activities

Have students report on the following:
Horner's method of approximating irrational roots
algebraic solutions of the cubic and quartic
history of the theory of equations.

References

Basic and auxiliary textbooks

Suggested Time

Three weeks

XVI. INEQUALITIES

A. Basic Concepts

1. Ordering of the real numbers
2. Absolute value of a real number
3. Basic rules for inequalities

B. Absolute inequalities

1. The inequality $a^2 + b^2 \geq 2ab$
2. The law of the mean

$$\frac{a+b}{2} \geq \sqrt{ab}$$

Objectives

To study the order properties of real numbers

To prove several types of absolute inequalities, and apply the results to maxima or minima problems

To solve conditional inequalities with one and two unknowns

To analyse the graphs of inequalities

Teaching Suggestions

Inequalities are important in both pure and applied

3. Applications to maxima and minima problems

C. Solution of Conditional Inequalities

1. Linear inequalities in one variable
2. Quadratic inequalities in one variable
3. Simultaneous inequalities in two variables
4. Graphical methods

mathematics. In addition, the solution of inequalities offers opportunities for analytical thinking. If the students are familiar with the set concept, the solutions of inequalities may be expressed in set notation.

Related Activities

Have students report on the following:
absolute inequalities not given in text
conditional inequalities above the second degree
application of inequalities to linear programming.

References

Kemeny, Snell, and Thompson. Introduction to Finite Mathematics, chapter VI.

Basic and auxiliary textbooks

Suggested Time

Two weeks

XVII. STATISTICS (Optional)

Objectives

To understand the vocabulary used in statistics

To learn techniques for collecting and classifying statistical data

To represent statistical data in graphical form

A. Recording of Statistical Data

1. Gathering of data and random sampling
2. Frequency distribution and use of frequency tables and class intervals

3. Graphical representation

B. Measures of Central Tendency

1. The mode
2. The median
3. The arithmetic mean

C. Measures of Dispersion

1. Average deviation
2. Standard deviation
3. Range of distribution
4. Quartiles
5. Percentiles

D. Correlation

1. Meaning of correlation
2. Linear and nonlinear correlation
3. Coefficient of linear correlation
4. Curve-fitting and scatter diagrams

E. Binomial Distributions

1. Tables of normal distributions
2. Histogram for binomial distribution
3. The normal frequency curve
4. The skew frequency curve

Objectives

To derive formulas for measuring central tendency, dispersion, and correlation

Teaching Suggestions

Many problems of science, industry, and government involve the consideration of large masses of numerical data obtained by counting or by measurement. In order to make such data meaningful, statistics must deal with questions of sampling, classification, presentation, analysis, and inference. An excellent way of attaining the objectives of this unit is to introduce a significant statistical problem and carry through the program as outlined in this unit.

Related Activities

Have students report on the following:
statistical studies reported in newspapers and periodicals
use of probability in statistics
use of statistics in public opinion polls.

References

Basic and auxiliary textbooks

Suggested Time

One week

XVIII. MATHEMATICAL INDUCTION (OPTIONAL)

Objectives

A. Nature of Inductive Reasoning

To explain the method of proof called

1. Deductive versus inductive reasoning
2. Inductive reasoning in science
3. Induction and plausible reasoning in mathematics

B. Principle of Mathematical Induction

1. Axiom of mathematical induction
2. The three parts of a proof by mathematical induction:
   a. verification
   b. proof of the inductive property
   c. conclusion
3. Proof of identities by mathematical induction
4. Proof of properties by mathematical induction
5. Proof of the binomial theorem by mathematical induction

mathematical induction

To prove the validity of identities involving n for all natural values of n

To prove properties of mathematical expressions involving n

Teaching Suggestions

Up to now, the student is familiar only with deduction as a technique in proving propositions. The principle of mathematical induction is based upon the axiom that if a proposition involving the positive integer n can be proved to have the following properties:

(a) the proposition is true for n = 1, and

(b) when k is any value of n for which the proposition is true, if then the proposition is also true for the next value n = k + 1,

then the proposition is true for all positive intergral values of n.

Illustrate the method of mathematical induction in several simple problems. Cite also examples in which the method fails because either part (a) or part (b) of the axiom is not satisfied.

Related Activities

Have students report on formulas and relations from geometry, algebra, and trigonometry which can be proved by mathematical induction.

References

Basic and auxiliary textbooks

Suggested Time

One week

TRIGONOMETRY

This course, in keeping with the present trend, places an emphasis on analytical concepts and is planned for students who are interested in mathematics and the sciences. It may be elected by any student who has successfully completed Algebra III and who has sufficient mathematical background to profit by further work in college preparatory mathematics. The present trend is to teach Algebra III and Trigonometry in the 11th grade.

However, to meet the needs of students in schools which cannot offer Trigonometry, Algebra IV, and Solid Geometry every semester, these subjects in grades 11A through 12A may be elected in any order.

A special course in practical trigonometry is given for students not planning to continue with the regular mathematics sequence but needing some knowledge of trigonometry for possible use in the world of work.

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I. TRIGONOMETRIC FUNCTIONS OF ANY ANGLE

A. Review of the Basic Concepts of Angles

1. Angles with terminal side in each of the four quadrants
2. Angles generated in counter-clockwise and clockwise directions
3. Understanding of degrees and radians

B. Meaning of the Six Trigonometric Functions

1. Definitions of the functions
2. Signs of the functions
   a. Representation of the six trigonometric functions of an acute angle, as line segments, using a unit circle
   b. Representation of the six trigonometric functions of angles with terminal sides in second, third, and fourth quadrants, as line segments, using a unit circle
3. Functions of the quadrantal angles
4. Derivation of the values of the functions of the special angles, $30^{\circ}, 45^{\circ}, 60^{\circ}$

C. Applications of the Trigonometric Functions

Objectives

To review the basic geometric concepts necessary for an understanding of the trigonometric functions

To learn the trigonometric functions

Teaching Suggestions

Introduce the unit with a brief review of angle concepts. The study of the trigonometric functions and related concepts should be thoroughly mastered in order that the remaining units of the course may be successfully studied.

Related Activities

Have students report on the following:
history of the development of trigonometry
careers requiring a knowledge of trigonometry.

View film Periodic Functions.

References

Yates. "The Trigonometric Functions," The Mathematics Teacher. Vol. LI, No. 3 (March, 1958), pp. 191-193.

Basic and auxiliary textbooks

Suggested Time

Three weeks

1. Graphing of an angle when one function is known
2. Finding of all the functions when one is known
3. Reduction of the functions of angles greater than 90^{o} to the functions of an acute angle
4. Finding of the values of functions of an acute angle
5. Reciprocal functions
6. Determinations of the functions of complementary angles
7. Pythagorean relations of functions
8. Using the table of natural functions to graph the six trigonometric functions for the interval, -360^{o} to $+360^{o}$
9. Solution of right triangles by natural functions

II. LOGARITHMS

A. Review of Scientific Notation, Approximate Numbers, and Rounding Off Numbers in Computation

1. Properties of exponents
2. Applications in the solution of problems

B. Meaning of Logarithms

1. Common logarithms, base 10
2. Logarithms with other bases
3. Characteristic and mantissa
4. Antilogarithms

Objectives

To understand the meaning of logarithms and how to use them

To learn how to use a table of logarithms

To use logarithms in solving equations and problems

Teaching Suggestions

The time required for a review of the properties of exponents, scientific notation, approximate numbers, and significant figures will depend upon the needs of the class. If students have taken Algebra IV before Trigonometry, Sections B, C, and F will also be review work, leaving only Sections D and E as advanced work.

C. Use of Tables of Logarithms

1. Logarithms of numbers in tables
2. Interpolation to find logarithms of numbers not in tables
3. Interpolation to find anti-logarithms
4. Computations involving logarithms

D. Use of Tables of Logarithms of the Trigonometric Functions

1. Finding of a logarithm of a trigonometric function directly from the table and by interpolation
2. Finding of the angle given the logarithm of one of its trigonometric functions

E. Computations Using Logarithms of the Trigonometric Functions

F. Solution of Exponential Equations

G. Explanation of Slide Rule (Optional)

Students should have enough practice to become proficient in the use of the tables of logarithms and to use logarithms effectively in the solution of trigonometric problems.

If slide rules are available, the use of these instruments in finding powers, roots, trigonometric functions, and logarithms should be explained. Students may use the slide rule to check work later in the course.

Related Activities

Have students report on the works of John Napier, Edmund Gunter, and Henry Briggs.

References

Seebeck. "The Logarithm Function to the Base e," The Mathematics Teacher, Vol. LIV, No. 1, (January, 1961), pp. 2-3.

Basic and auxiliary textbooks

Suggested Time

Two weeks

III. SOLUTION OF RIGHT TRIANGLES

A. Use of Logarithms in the Solution of All Cases of Right Triangles

1. Finding of area of right triangles
2. Solution of isosceles triangles

Objective

To apply the concepts learned in Units I and II to the solution of practical problems involving right triangles

Teaching Suggestions

This unit should offer a wide variety of practical problems

B. Solution of Practical Problems Involving Right Triangles

1. Navigation problems
2. Surveying problems
3. Distance problems
4. Heights of objects

which will add interest to the class work. The pocket transit or simplified transit will be very helpful in solving these practical problems. The use of the slide rule will shorten the working time of problems. It may also be used as a check for the problems solved by logarithms.

Related Activities

Have students report on the use of the simple transit and surveying instruments.

View film Indirect Measurement.

References

Shuster and Bedford. Fieldwork in Mathematics.

Basic and auxiliary textbooks

Suggested Time

Two weeks

IV. FUNCTIONS OF TWO ANGLES

Objectives

To obtain a basic knowledge of some of the important trigonometric formulas

To apply these formulas to the solution of problems

A. Development of the Formulas for the Sine, Cosine, Tangent, and Cotangent for the Sum of Two Angles and for the Difference of Two Angles

B. Applications of the Sum and Difference Formulas

C. Derivation of the Formulas for Sine,

Teaching Suggestions

The development of the formulas for the sine and cosine of the sum of two angles offers an opportunity to review

Cosine, and Tangent of Twice an Angle

D. Derivation of the Formulas for Sine, Cosine, and Tangent of Half an Angle

E. Derivation of the Product and Sum Formulas

F. Solution of Trigonometric Equations

some of the basic theorems in plane geometry. Review these theorems before developing the formulas.

Related Activities

Have students report on the history of the development of the formulas in this unit.

References

Basic and auxiliary textbooks

Suggested Time

Three weeks

V. IDENTITIES AND EQUATIONS

A. Meaning of Identity and Equation

1. Derivation of the eight fundamental identities
2. Finding of equivalent trigonometric expressions

B. Solution of Identities and Equations Involving Single Angles

C. Solution of Identities and Equations Involving Two Variables

Objectives

To understand the difference between an identity and equation

To solve trigonometric identities and equations

Teaching Suggestions

Proving identities may be a new experience for some students. Simple identities should be given until the students understand the method of solution.

Related Activities

Have students bring to class and post on the bulletin board problems in identities and trigonometric equations.

Discuss with the class the applications of identities in higher mathematics, particularly the calculus.

VI. SOLUTION OF OBLIQUE TRIANGLES

A. Derivation of the Law of Sines

1. Application of the law of sines to solving a triangle, given a side and two angles
2. Application of the law of sines to solving a triangle, given two sides and an angle opposite one of them.

B. Derivation of the Law of Cosines

1. Application of the law of cosines to solving a triangle, given two sides and the included angle
2. Application of the law of cosines to solving a triangle, given three sides

C. Derivation of the Formulas for the Areas of Oblique Triangles

1. Two sides and the included angle known
2. Hero's Formula, three sides known

References

Basic and auxiliary textbooks

Suggested Time

One week

Objective

To learn the laws and formulas of trigonometric functions used to solve oblique triangles

Teaching Suggestions

Review the determination of a triangle and the congruence theorems. Show how these basic concepts are used in the applications of the law of sines and the law of cosines. Review the Pythagorean relations needed in deriving Hero's formula.

Related Activities

Have students report on the following:

solution by trigonometry of the geometric theorems for the square of the side opposite an acute angle of a triangle, and the square of the side opposite an obtuse angle of a triangle

the use of the triangle of velocities in air navigation

the use of the parallelogram of velocities in aviation.

D. Derivation of the Law of Tangents

E. Derivation of the Half-angle Formulas

F. Solution of Practical Problems

References

Clifford. "An Application of the Law of Sines," The Mathematics Teacher, Vol. LIV, No. 5 (May, 1961) pp. 346-350.

Basic and auxiliary textbooks

Suggested Time

Three weeks

VII. RADIAN AND MIL MEASURE

Objectives

A. Definition of Radian
   1. Relation between radian and degree
   2. Application to finding the length of an arc of a circle
   3. Application to finding areas of sectors and segments of circles
   4. Application to circular motion

B. Definition of Mil
   1. Relation between mil and degrees, and mil and radians
   2. Use of mil in measuring angles

To understand the meaning of radian and mil

To apply this understanding to the solution of problems

Teaching Suggestions

The practical applications of the use of radians and mils should be emphasized in this unit.

Related Activities

Have students make reports on the following:
use of the mil in the Armed Forces
practical uses of the radian
linear and angular speed-revolutions per second.

References

Landon. "Mechanics of Orbiting," The Mathematics Teacher, Vol. LII, No. 5 (May, 1959) pp. 361-364.

Basic and auxiliary textbooks

Suggested Time

One week

VIII. CHARACTERISTICS AND GRAPHS OF THE TRIGONOMETRIC FUNCTIONS

A. Brief Review of Graphs of Algebraic Functions

B. Review of Line Values of the Trigonometric Functions in the Four Quadrants

C. Understanding of the Meaning of Slope of a Line and Slope of a Curve

D. Understanding of the Importance of Periodic Functions

E. Graphing of the Six Trigonometric Functions

F. Graphing of Trigonometric Functions Consisting of Sums (Optional)

Objectives

To review some of the analytic properties of graphs

To show the periodicity of the trigonometric functions graphically

Teaching Suggestions

Have students use both rectangular and polar coordinate graph paper.

Emphasize the period of the function in each of the six functions.

Related Activities

Have students sketch graphs of trigonometric relations on polar coordinate paper.

$$r = \frac{1}{1 - \cos \theta}$$

$$r = \sin 3\theta$$

$$r = \cos 2\theta$$

$$r = 1 - 2 \cos \theta$$

References

Allendoerfer and Oakley. Principles of Mathematics.

Basic and auxiliary textbooks

Suggested Time

Two weeks

IX. INVERSE TRIGONOMETRIC FUNCTIONS

A. Meaning of Inverse Functions

1. Principal values of inverse functions
2. Graphs of inverse functions

B. Solution of Identities Involving Inverse Functions

C. Solution of Equations Involving Inverse Functions

Objective

To learn the meaning of inverse functions

Teaching Suggestions

Show how the word "inverse" has different meanings according to the way it is used. Some of the common usages in mathematics are additive inverse, multiplicative inverse, and inverse function. In each case the inverse is the opposite of the operation.

Related Activities

Have students report on the relationship between logarithms and the circular and hyperbolic trigonometric functions.

References

Basic and auxiliary textbooks

Suggested Time

Two weeks

X. RECTANGULAR AND POLAR REPRESENTATION OF COMPLEX NUMBERS (OPTIONAL)

A. Review of Complex Numbers

1. Definition
2. Rectangular Representation
3. Operations with Complex Numbers

B. Polar Representation of Complex Numbers

1. Polar form of a complex number
2. Multiplication and division of complex numbers in polar form

C. De Moivre's Theorem

Objectives

To review rectangular representation of complex numbers

To acquire an understanding of polar representation of complex numbers

Teaching Suggestions

The review of complex numbers should be brief but adequate. Show the relation between the rectangular form and polar form of a complex number by graphing the number in rectangular form. Then show how the same point may be represented in polar form.

Related Activities

Have students make graphs for display on the bulletin board.

Discuss with the students applications of De Moivre's Theorem.

References

Basic and auxiliary textbooks

Suggested Time

One week

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PRACTICAL TRIGONOMETRY

Practical trigonometry is designed for students who have completed at least one year of algebra and one year of geometry, and for those who wish to enter employment requiring the use of trigonometry. It is not a college-preparatory course. Students who wish to major in mathematics, or who have been in honors mathematics courses, should register for the regular or honors course in trigonometry.

This course emphasizes the many applications of trigonometry and includes enough theory to make the basic concepts understandable. The concepts of logarithms and slide rule are given early in the course to enable the students to become proficient in the use of these skills and thereby shorten the working time for solving problems.

Trigonometry is related to arithmetic, algebra, and geometry. These relationships should be used to introduce new concepts. The graphing of some of the trigonometric functions is given in the first unit rather than near the end of the course. This will allow more time for practical applications later in the course.

PRACTICAL TRIGONOMETRY

I. TRIGONOMETRIC FUNCTIONS OF ANY ANGLE

A. Review of Basic Definitions of Angles

1. Explanation of initial side, generating or terminal side
2. Explanation of angles generated in counter-clockwise and clockwise directions
3. Units of measurement
   a. Degrees
   b. Radians

B. Definition of the Six Trigonometric Functions

1. Signs of the trigonometric functions for angles in each of the four quadrants
2. Quadrantal angles
3. Graphing of an angle when one function is known
4. Finding of the other functions when one function is known
5. Reciprocal functions
6. Relations of functions of complementary angles
7. Functions of 30°, 45°, and 60° angles
8. Use of table of natural functions

Objectives

To review the concepts of angles and similar triangles studied in plane geometry

To apply these concepts to the study of trigonometric functions

Teaching Suggestions

The students taking this course may have had only four semesters of high school mathematics. The presentation of the new concepts should be taught from a practical rather than a theoretical viewpoint. The signs of the functions in each of the four quadrants can be taught by the unit circle with its center at the origin of the graph. A geometric representation of the functions by their graphs for the interval 0° to $+ 360^{\circ}$ should give a better appreciation for these concepts.

Related Activities

Have students represent the trigonometric functions as line segments on a peg board. Use golf tees and round elastic or colored rubber bands.

Discuss with the students the history of the development of trigonometry.

Have students report on careers that require a knowledge of trigonometry.

C. Graphing of the Sine, Cosine, and Tagent Functions for Interval 0^{o} to + 360^{o}

References

Basic and auxiliary textbooks

Suggested Time

Three weeks

II. LOGARITHMS

Objectives

To review the meaning of approximate numbers

To obtain an understanding of logarithms and ability to work with logarithms

A. Meaning of Approximate Numbers

1. Rounding off of numbers
2. Scientific notation
   a. Properties of exponents
   b. Computation with scientific notation

B. Meaning of Logarithms

1. Logarithms with different bases
2. Common logarithms, base 10
3. Characteristic and mantissa of common logarithms
   a. Finding of logarithms of numbers
   b. Finding of antilogarithm

C. Computation with Logarithms

1. Finding of logarithms by interpolation
2. Finding of antilogarithms by interpolation

Teaching Suggestions

The concepts of approximate number, scientific notation, and the properties of exponents should be reviewed until the students can use them with ease. Explanation of logarithms will then follow naturally. Enough time should be used in the explanation of the meaning of logarithms so that students thoroughly understand the basic concepts before computing with them.

Related Activities

Have students make reports on the following:
- scientific notation and its uses in computing machines
- John Napier, Edmund Gunter, and Henry Briggs

3. Computations involving interpolation

D. Solution of Exponential Equations

E. Finding of Logarithms of the Trigonometric Functions

F. Finding of the Angle When the Logarithm of One of Its Functions Is Known

G. Computations Involving Logarithms of the Trigonometric Functions

References

Basic and auxiliary textbooks

Suggested Time

Three weeks

III. SLIDE RULE

A. Explanation of the Use of Logarithms in Computations by the Slide Rule

1. Parts of the slide rule
2. Logarithmic scale
3. Reading of logarithmic scale

B. Multiplication with a Slide Rule

C. Division with a Slide Rule

D. Squaring of a Number with a Slide Rule

E. Extraction of the Square Root of a Number with a Slide Rule

F. Solution of Problems

Objective

To learn to use the slide rule accurately in computations

Teaching Suggestions

Use of the slide rule will shorten the time of problem solving. The students should have enough practice with this device so that they can use it with ease. Demonstration slide rules have been obtained for the high schools through funds from the Chicago Board of Education and funds from the N.D.E.A., Title III Program. These instruments should be used for this unit.

Related Activities

Have the students report on the history of the slide rule.

Have students form a slide rule club.

Show the following motion pictures:

Slide Rule: Multiplication and Division

Slide Rule: Percentage, Proportion, Squares, and Square Root.

References

Welchons and Krickenberger. Trigonometry with Tables, 1957 Edition, pp. 89-103.

Welchons, Krickenberger and Pearson. Algebra, Book Two, 1960 Edition, pp. 313-320.

Other basic and auxiliary textbooks

Suggested Time

Two weeks

IV. SOLUTION OF THE RIGHT TRIANGLE

A. Use of Table of Natural Functions in Solution of Problems Involving Right Triangles

B. Use of Table of Trigonometric Functions in Solution of Problems Involving Right Triangles

1. Only sine, cosine, tangent, and cotangent functions
2. Slide rule to check computations

Objective

To apply the concepts of logarithm and slide rule to the solution of problems

Teaching Suggestions

The students should have enough practice in the solution of right triangles to enable them to use the tables of natural functions and trigonometric functions with ease.

C. Solution of Right Triangle Given One Side and an Acute Angle

D. Solution of Right Triangle Given Two Sides

A minimum amount of time should be used for this unit.

Related Activities

Have students make reports on the following:
use of angles in surveying
calculation of the resultant of two forces acting simultaneously at right angles to each other.

References

Basic and auxiliary textbooks

Suggested Time

Two weeks

V. SOLUTION OF OBLIQUE TRIANGLES

A. Law of Sines

B. Law of Cosines

C. Law of Tangents (Optional)

D. Area Formulas

E. Solving of Problems Involving Oblique Triangles

Objectives

To understand the law of sines and the law of cosines and to know how to use them

To apply the laws and formulas in this unit to the solution of problems

Teaching Suggestions

Many of the applications of trigonometry involve oblique triangles. The students should understand the applications of the principles of right triangles to oblique triangles.

Related Activities

Have students make reports on the following:
the use of the triangle of velocities in air navigation
the use of the parallelogram of velocities in solving problems in aviation
the resultant of two forces acting simultaneously at an angle other than 90°.

References

Clifford. "An Application of the Law of Sines," The Mathematics Teacher, Vol. LIV, No. 5 (May, 1961), pp. 346-350.

Griswold, Keedy, and Schacht. Contemporary Algebra and Trigonometry, pp. 364-374.

Basic and auxiliary textbooks

Suggested Time

Three weeks

VI. RADIAN AND MIL MEASURE

A. Definition of Radian Measurement

B. Relationship between Radian and Degree

C. Use of Tables of Radians and Degrees

Objective

To understand the meaning and application of radian and mil measurement

Teaching Suggestions

Radian and mil measurements are becoming increasingly important in our everyday life.

D. Finding of the Length of an Arc of a Circle

E. Finding of the Areas of Sectors and Segments of Circles

F. Definition of Mil Measure

1. Relationship between mils and degrees, and mils and radians
2. Application of mils to solving problems, as computing firing data, sketching, and mapping

Students should have a basic understanding of the meaning of these measures.

The skill of changing from degrees to radians to mils, in any order, should be taught before solution of problems involving radians and mils.

Related Activities

Have students discuss the practical uses of the radian and the use of the mil in the Armed Forces.

References

Basic and auxiliary textbooks

Suggested Time

Two weeks

VII. PRACTICAL APPLICATIONS

A. Solution of Problems by Use of the Simple Transit and Surveying Instruments

1. Field trips in the vicinity to record data
   a. Heights of buildings
   b. Distances to buildings and other objects
   c. Angle of elevation

Objectives

To teach the practical application of trigonometry

To have students obtain practice in using surveying instruments

Teaching Suggestions

This unit suggests some practical problems. The textbooks have many more for the students to

2. Field trips by one or more students to record data to compute the heights of well-known buildings in the city or community

B. Solution of Problems in Navigation, Surveying, Aviation, and Physics

solve. The use of the simple transit and the surveying instruments will stimulate the students and add interest to the unit.

Related Activities

Have the students make lists of practical problems and have them solved in class.

References

Landon. "Mechanics of Orbiting," The Mathematics Teacher, Vol. LII, No. 5 (May, 1959), pp. 361-364.

Shuster and Bedford. Fieldwork in Mathematics.

Basic and auxiliary textbooks

Suggested Time

Four weeks

Review and Tests

One week

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SOLID GEOMETRY

The regular college preparatory course in plane geometry correlates only a few of the concepts of three-dimensional space with the plane geometry concepts. Because of this fact, and the fact that we live in a three dimensional world, in which space relations are so important in all aspects of modern living, the High School Evaluation Committee feels that a course in solid geometry should be retained in the mathematics program at this time. This course affords the student an opportunity to learn the properties of some of the familiar solids which are a part of his world. It also helps him to retain and extend the logical reasoning techniques which he learned in plane geometry.

Although this course usually is one of the mathematics electives in the 12th grade, it may be made available to any student who has completed the courses in plane geometry and Algebra III, and who has sufficient interest and mathematical background to profit by a study of solid geometry.

The outline presents the unit on polyhedrons, followed by a unit on cylinders and cones. Some teachers may prefer to teach cylinders with prisms and cones with pyramids. If so, they should feel free to rearrange the order of these units.

Although the course should be taught in a rigorous manner, many of the theorems and corollaries may be class demonstrations with individual work centered on originals and practical applications.

This is a standard course in solid geometry, but teachers may supplement it and enrich it wherever possible with concepts from modern mathematics. It is hoped that this will be the general procedure.

REGULAR SOLID GEOMETRY

I. INTRODUCTION

A. Relationship of Plane and Solid Geometry

B. Basic Concepts

1. Point
2. Line
3. Plane
4. Surface
5. Space
6. Solid

C. Familiar Solids

D. Conditions Which Determine a Plane

E. Postulates and Preliminary Theorems

F. Perspective Drawing

Objectives

To understand the relationship of plane and solid geometry

To gain some skill in visualizing spatial relations

To acquire understanding of some of the basic concepts of solid geometry

To develop skill in perspective drawing

Teaching Suggestions

If students have not studied geometry for a year or more, it may be necessary to review some of the basic concepts of plane geometry. Provision is made for most of these concepts to be reviewed and used as the work in solid geometry progresses.

It might be well to remind students that a mathematical system consists of undefined terms, defined terms, postulates, and theorems, and to review the nature of geometric proof.

Provide sufficient practice in perspective drawing so that each student acquires the ability to represent three-dimensional figures in a plane.

Related Activities

Use the large models of solids provided through the Board of Education and NDEA to discuss familiar solids.

Have students prepare a bulletin board display of perspective drawings.

Use the opaque projector to show some of the better perspective drawings made by students.

Recall the meaning of converse, inverse, and contrapositive.

View film Understanding Solids in Geometry.

References

Basic and auxiliary textbooks

Suggested Time

One week

II. LINE AND PLANE RELATIONSHIPS

A. Perpendicular Lines and Planes

1. Perpendicular lines in space
2. Perpendicular planes and lines in space

B. Lines Oblique to a Plane

C. Parallel Lines and Planes

1. Lines parallel to planes
2. Perpendiculars forming parallels
3. Intersecting planes

Objectives

To understand the conditions under which lines and planes are perpendicular

To understand the relationships of parallel lines and planes

Teaching Suggestions

Review the meaning of parallel, perpendicular, oblique, coincide, collinear, plane, coplanar, and other terms used in the unit.

Remind students that the axioms, postulates, and theorems of plane geometry may be used in proving the theorems of solid geometry.

Have pupils recall theorems in plane geometry which are analogous to those in this unit.

Related Activities

Have students make models to use in demonstrating the theorems of the unit.

Discuss the many uses of solid geometry in art, architecture, industry, and science.

References

Basic and auxiliary textbooks

Suggested Time

Two weeks

III. DIHEDRAL AND POLYHEDRAL ANGLES

A. Dihedral Angles

1. Meaning of dihedral angle
2. Plane angle of a dihedral angle
3. Measure of a dihedral angle
4. Kinds of dihedral angles
5. Proofs of line and plane relationships involving dihedral angles

B. Polyhedral Angles

1. Meaning of polyhedral angle
2. Kinds of polyhedral angles
3. Congruent and symmetric polyhedral angles
4. Proofs of theorems involving polyhedral angles

Objectives

To understand what a dihedral angle is and how to read it and measure it

To understand the properties of polyhedral angles

To prove some of the geometric relationships involving dihedral and polyhedral angles

Teaching Suggestions

Discuss angles formed by two lines, a line and a plane, two planes, and by three or more planes meeting at a common point.

Illustrate dihedral angles by an open book or by a folded sheet of paper.

Call attention to dihedral angles and polyhedral angles in a set of geometric solids, in the classroom, and in other buildings in the community.

Related Activities

Have students explain why longitude is measured by a dihedral angle.

Discuss use of dihedral angles in aeronautics.

References

Basic and auxiliary textbooks

Suggested Time

Two weeks

IV. LOCI AND PROJECTIONS

A. Loci

1. Loci as sets of points
2. Review of fundamental locus theorems of plane geometry
3. Locus theorems and problems
4. Intersecting loci

Objectives

To use locus concepts and problems to increase space thinking

To understand various types of projection

Teaching Suggestions

The study of loci in solid geometry provides another opportunity to broaden and deepen this concept, which is so important in the study of mathematics beyond the high school. Present some of the familiar locus problems of plane geometry and then discuss the corresponding problems in solid geometry.

B. Projections

1. Meaning of projection
2. Parallel projection
3. Central projection

4. Orthogonal projection
5. Projection of points and lines on planes
6. Projection theorems and problems

Develop the meaning of projection with models using thread, string, and wire.

Related Activities

Show the motion picture Locus, and follow the showing with a class discussion.

Have pupils bring to class projections, such as photographs and aerial maps.

Use the opaque projector to project pictures, drawings, and charts on a screen and discuss this type of projection with the class.

Have students show how to find the areas of projections by using one of the trigonometric ratios.

Discuss with the class some of the applications of projective geometry.

References

Basic and auxiliary textbooks

Suggested Time

Two weeks

V. POLYHEDRONS

A. Parts of Polyhedrons
B. Names of Polyhedrons

Objectives

To learn the properties of the various polyhedrons

To acquire the ability to find surface areas and volumes of polyhedrons

C. Prisms

1. Parallelopipeds
2. Surface area of prisms
3. Rectangular solids
4. Volume of a solid
5. Cavalieri's principle
6. Volume of a prism

D. Pyramids

1. Surface area of a pyramid
2. Truncated pyramids
3. Areas and volumes of pyramids and frustums of pyramids

E. Similar Polyhedrons

F. Regular Polyhedrons

To appreciate and learn some of the uses of polyhedrons in art, architecture, engineering, industry, and science.

Teaching Suggestions

Show models of the various polyhedrons. Most of these are in a set furnished to each high school by the Board of Education and NDEA. Allow students to handle the models and discuss the parts of the various polyhedrons. Have them make paper patterns of the solids in order to see the lateral and total surfaces. Develop area formulas from these paper patterns. The formulas developed in this unit have many practical applications. Give students as much practice on these applications as time will permit.

Related Activities

Have students report on:
the use of polyhedrons in art and architecture
the history of the pyramids of Egypt
the history and the special properties of the five regular polyhedrons.

Have students make:
models of the regular polyhedrons
models to illustrate Cavalieri's theorem.

Discuss with the class efforts to "duplicate the cube."

Demonstrate how to measure the volume of a solid of irregular shape by measuring the volume of a liquid it contains or displaces.

Mount pictures of applications of polyhedrons in

industry and science, and project these on the screen with the opaque projector.

Discuss Euler's Theorem, $e + 2 = v + f$, when the number of edges is denoted by e, the number of vertices by v, and the number of faces by f.

Show how to find the volume of a prismatoid.

View film Discovering Solids: Volumes of Cubes, Prisms, and Cylinders.

References

Basic and auxiliary textbooks

Suggested Time

Four weeks

VI. CYLINDERS AND CONES

A. Cylinders

1. Parts of cylinders
2. Kinds of cylinders
3. Areas of cylinders
4. Volumes of cylinders
5. Practical applications
6. Similar cylinders of revolution

B. Cones

1. Parts of cones
2. Kinds of cones
3. Areas of cones

Objectives

To learn the properties of cylinders and cones

To acquire the ability to find surface areas and volumes of cylinders, cones, and frustums of cones

To review the properties of the conic sections

Teaching Suggestions

Develop the new concepts carefully at the beginning of the unit. As the work of the unit progresses, show the relation of the formulas for the lateral area and volume of a cylinder to the corresponding formulas of the prism.

4. Volumes of cones
5. Areas and volumes of frustums of cones
6. Similar cones
7. Conic sections
8. Practical applications

Also relate the formulas for the lateral area and volume of a cone to those of a pyramid. Use models furnished through the Board of Education and NDEA to develop concepts of cylinders, cones, and the conic sections.

Related Activities

Discuss the center of gravity of cylinders and cones.

Have students report on interesting facts about the Leaning Tower of Pisa.

Discuss the theory of limits.

View film Oblique Cones and Transitional Developments.

References

Basic and auxiliary textbooks

Suggested Time

Four weeks

VII. SPHERES AND SPHERICAL POLYGONS

Objectives

To learn the properties of spheres

To understand the properties of spherical polygons

To acquire facility in computing spherical surfaces and volumes

A. Spheres

1. Parts of spheres
2. Circles of spheres
3. Polar distance
4. Tangents to a sphere
5. Inscribed and circumscribed spheres

B. Spherical Polygons

1. Spherical angles
2. Spherical triangles
3. Polar triangles
4. Congruent spherical polygons
5. Symmetric spherical polygons

C. Spherical Areas

1. Area of a sphere
2. Area of a zone
3. Area of a lune
4. Spherical excess
5. Area of a spherical triangle
6. Applications

D. Spherical Volumes

1. Volume of a sphere
2. Volume of a spherical pyramid
3. Volume of a spherical sector
4. Volume of a spherical cone
5. Volume of a spherical wedge
6. Volume of a spherical segment
7. Applications

Teaching Suggestions

Use the slated globes and other models furnished to all high schools by the Board of Education and NDEA in teaching the concepts of this unit. The slated globe will be helpful in teaching such concepts as great circle, arc of a great circle, spherical distance, polar distance, spherical triangle, spherical polygon, lune, and zone. Use the models cone, sphere, and cylinder to demonstrate the relative volumes of the three when the height and diameter of the bases of the cylinder, height and diameter of the base of the cone, and diameter of the sphere are all the same.

Relate the properties of the sphere to those of the circle.

Call attention to the applications of this unit to air routes, time zones, latitude, and longitude.

Related Activities

Have students report on:
principal air routes of the world
the work of Archimedes on the sphere and cylinder
the measurement of the circumference of the earth by Eratosthenes
the meaning of celestial sphere.

Discuss some of the applications of the unit to the space age.

Using a hemisphere, have students make a model showing that the surface area of a sphere is equal to four times the area of one of its great circles.

Have students circumscribe solids about a ball or wooden sphere and explain the relationships.

References

Basic and auxiliary textbooks

Suggested Time

Five weeks

The Natural Numbers

The number system developed as man needed some way to keep records of trading and other business transactions with his neighbors. The symbols 1, 2, 3,... which we use today are called the natural number system or the set of natural numbers. Note that the set of natural numbers, when considered as counting numbers, has no zero symbol. In early times there was no need for such a symbol.

As practical problems arose, addition and multiplication, which is repeated addition, developed naturally. The natural numbers were adequate for these operations.

The sum of two natural numbers is a natural number and the product of two natural numbers is a natural number. When an operation is performed on two numbers, of a set of numbers, and the result is a number of that set, the operation is said to have closure. In the set of natural numbers, consider the statement $3 + 4 = 7$. The sum of the natural numbers 3 and 4 is 7, a natural number. Now consider $3 \times 4 = 12$. The product of the natural numbers 3 and 4 is 12, a natural number. Thus, closure is preserved with respect to addition and multiplication in the set of natural numbers.

When the operations of subtraction and division were performed with natural numbers, the results were not always natural numbers. For example, $4 - 5 = -1$ and $4 \div 5 = \frac{4}{5}$. These results are not natural numbers. Thus closure is not preserved.

The Integers

To preserve closure in subtraction the set of natural numbers was extended to include zero and negative numbers. The set of natural numbers with the addition of zero and negative numbers formed the set of integers. In the set of integers the operation of subtraction had closure.

The Rational Numbers

Maintaining closure in division in the set of integers led to the invention of the rational number system. A rational number is one which can be expressed as the quotient of two integers, as $\frac{a}{b}$, with b not zero. The collection of all such pairs of numbers is called the rational number system. The rational number system or the set of rational numbers, includes the set of integers and the set of natural numbers.

Irrational Numbers

The number system was again extended when man learned to extract roots (evolution). The roots of some positive rational numbers were positive rational numbers, while the roots of other positive natural numbers were not rational numbers. These roots were called irrational numbers. Examples of irrational numbers are $\sqrt{2}$, $\sqrt{5}$, $\sqrt{17}$.

Real Numbers

The set of irrational numbers with the set of rational numbers form the system of real numbers or the set of real numbers.

Properties of Equality of Real Numbers

Reflexive property

$a = a$

Symmetric property

If $a = b$, then $b = a$

Transitive property

If $a = b$, and $b = c$, then $a = c$

Properties of Operations of Real Numbers

Closure property

If a and b are real numbers and $a + b = c$, c is also a real number. If $a \times b = ab$, ab is a real number.

Commutative property

$a + b = b + a$
$a \times b = b \times a$

Associative property

$a + b + c = (a + b) + c = a + (b + c)$
$a \times b \times c = a \cdot (b \times c) = (a \times b)\ c$

Distributive property

$a\ (b + c) = a \times b + a \times c = ab + ac$
$ab + ac = a\ (b + c)$

Identity property

$a + 0 = 0 + a = a$
$a \times 1 = 1 \times a = a$

Inverse property

$a \times \frac{1}{a} = \frac{1}{a} \times a = 1$, for $a = 0$

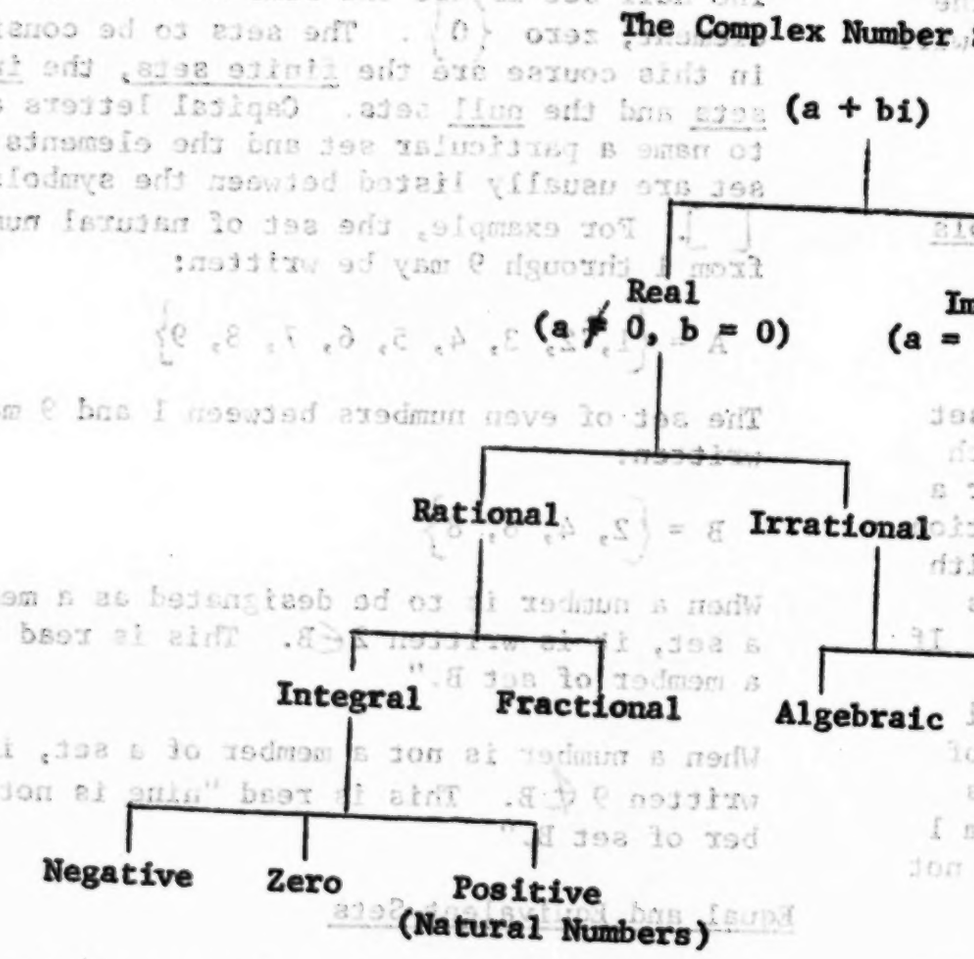
$a + (-a) = (-a) + a = 0$

Complex Numbers

The set of complex numbers include both algebraic and non-algebraic numbers. An algebraic number is one which may be the root of an algebraic equation, in which the coefficients of its terms are rational numbers and the exponents are positive integers. Irrational roots of such equations are algebraic numbers and form a very large set. In fact there are more irrational roots than rational roots in the ordinary problems.

Irrational numbers that are not roots of algebraic equations are called transcendental numbers. Examples of transcendental numbers, in common usage, are π and e.

The following diagram shows the relationship between the different kinds of numbers:


The Complex Number System
(a + bi)
Real
(a ≠ 0, b = 0)
Imaginary
(a = 0, b ≠ 0)
Rational
Irrational
Integral
Fractional
Algebraic
Transcendentals
Negative
Zero
Positive
(Natural Numbers)
Algebraic numbers
Non-algebraic
numbers

In the elementary algebra course, only the operations with the set of real numbers will be emphasized.

SETS

Basic Concept of Sets and Meaning of Symbols for Sets

Mathematical meaning of sets

The basic idea of set is not new. A set is simply a collection of objects, such as a set of dishes, a set of books, or a set of numbers. If the set or collection of dishes include only those dishes with a floral pattern, then all other types of dishes are not members of the set. If the collection of books includes only reference books in chemistry, then all other kinds of books are not members of the set. If the collection of numbers includes only the natural numbers from 1 through 9, then all other numbers are not members of the set.

Each object in the set is called a member or element of the set. A set may be very small or very large, depending on the number of its elements. If the number of elements of a set is countable, it is a finite set. If the number of elements of a set is not countable, it is an infinite set. If the set has no members, it is the empty set or null set. The empty set or null set is designated by the symbols $\emptyset$ or $\{\}$.

The null set is not the same as a set with one element, zero $\{0\}$. The sets to be considered in this course are the finite sets, the infinite sets and the null sets. Capital letters are used to name a particular set and the elements of the set are usually listed between the symbols $\{\}$ or $[\,]$. For example, the set of natural numbers from 1 through 9 may be written:

$$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

The set of even numbers between 1 and 9 may be written:

$$B = \{2, 4, 6, 8\}$$

When a number is to be designated as a member of a set, it is written $2 \in B$. This is read "two is a member of set B."

When a number is not a member of a set, it is written $9 \notin B$. This is read "nine is not a member of set B."

Equal and Equivalent Sets

The set of the first four letters of the alphabet may be written:

$$C = \{a, b, c, d\}$$

Note that set B has four members and set C also has four members. When two sets have the same number of members, they are said to be equivalent sets. In symbol form $C \leftrightarrow B$ and $B \leftrightarrow C$.

Let $D = \{8, 6, 2, 4\}$

Note that all the members of set B are also members of set D and both sets have the same number of elements. Sets B and D are called equal sets. Any two sets which have exactly the same members are equal sets. B = D.

If two sets are equivalent, each member of one set may be matched with a member of the equivalent set.

Since C$\leftrightarrow$B, the elements of C can be matched with the elements of B.

a$\leftrightarrow$2		a$\leftrightarrow$4
b$\leftrightarrow$4	or	b$\leftrightarrow$6
c$\leftrightarrow$6		c$\leftrightarrow$8
d$\leftrightarrow$8		d$\leftrightarrow$2

There are several ways in which the elements of C can be paired with the elements of B. Each time the elements of the two sets are paired, no elements in either set are left. When the elements in two sets can be matched or paired in this way, the sets are said to have a one-to-one correspondence.

Subsets

If every member of a set, A, is also a member of a set, B, then A is a subset of B. This is written: A$\subset$B. If B has more members than A, then A is a proper subset of B.

If every member of A is also a member of B, A$\subset$B, and if every member of B is also a member of A, B$\subset$A, then A = B. If this is true of a given set M, then M is a subset of itself, M$\subset$M. In general, every set is a subset of itself. Also, the null set or empty set is a subset of every set.

Universal Set

A set is understood to be a collection of objects which have some common characteristics. For example, set A may be the odd numbers from 1 through 11. A = {1, 3, 5, 7, 9, 11}. Set A is a subset of all the numbers {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and 11.} It is also a subset of the natural numbers. A larger set of a particular set is called the universal set or the universe and is designated by a capital U. In this example U = the natural numbers. All members of the infinite set {1, 2, 3, 4,...} are in this set.

Operations with Sets

Diagrams

Pictures of a set and its subsets, or its relation with other sets are called diagrams. A rectangle is usually used for the universal set and circles for the subsets.

Figure 1 represents all the members of the universal set.

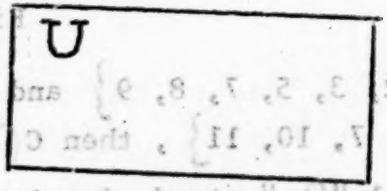


Figure 1

Figure 2 represents $A \subset U$. Set A is a proper subset of U.

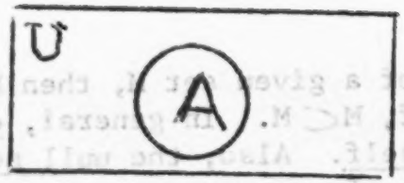


Figure 2

Figure 3 represents $A \subset U$ and $B \subset U$.

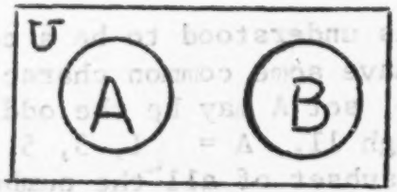


Figure 3

When two sets have no members in common, the sets do not intersect. Set A is <u>disjoint</u> from set B. Thus, the intersection of Set A and Set B is the null set.

Intersection and Union of Sets

Figure 4 represents the intersection of C and D. All the elements in the shaded part of the diagram contain the elements common to sets C and D.

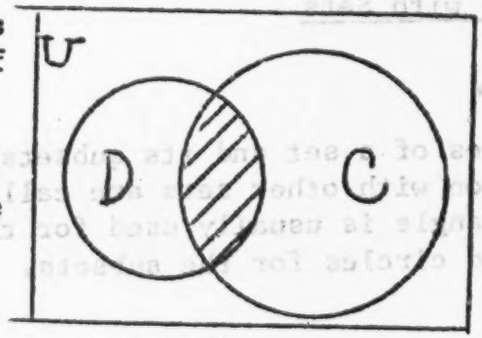


Figure 4

If $C = \{2, 3, 5, 7, 8, 9\}$ and $D = \{3, 5, 7, 10, 11\}$, then $C \cap B = \{3, 5, 7\}$.

The symbol "$\cap$" stands for <u>intersection</u>. It is also called <u>cap</u>.

In Figure 3, $A \cap B = \emptyset$ or $A \cap B =$

In Figure 5, the intersection of E and F is the set of one element.

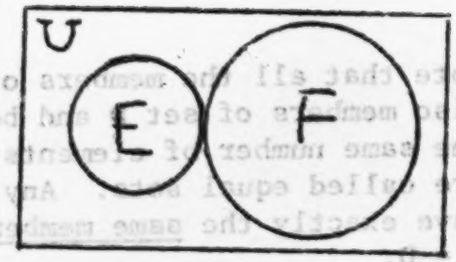


Figure 5

If $E = \{3, 4, 5\}$ and $F = \{1, 2, 3, 6, 7, 8\}$, then $E \cap F = \{3\}$.

In Figure 6, all the elements in U which are elements of at least one of the sets G or H form the set which is the <u>union</u> of G and H. The symbol $\cup$ is read <u>union</u>.

$G \cup H$ is the shaded portion of Figure 6.

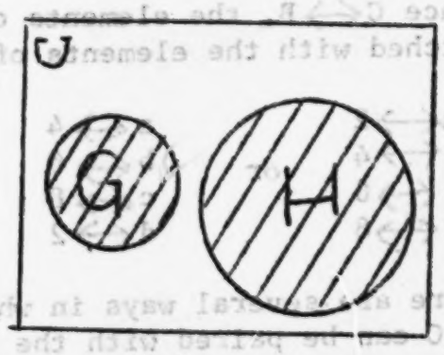


Figure 6

If $G = \{2, 4, 6\}$ and $H = \{3, 5, 7, 9\}$, then $G \cup H = \{2, 3, 4, 5, 6, 7, 9\}$. The sets are disjoint.

In Figure 7 if $K = \{1, 2, 3, 4, 5\}$ and $L = \{3, 4, 5, 8, 9, 10, 11\}$, then $K \cup L = \{1, 2, 3, 4, 5, 8, 9, 10, 11\}$

The sets intersect.

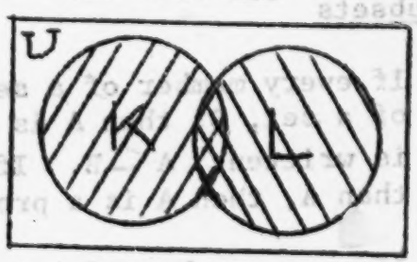


Figure 7

Note that the elements 3, 4, 5 are in both sets K and L, but they appear only once in the set formed by the union of sets K and L.

Figure 8 shows the intersection of sets M and N when N is a proper subset of M.

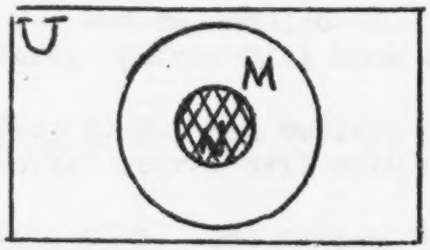


Figure 8

If $N \subset M$,
then $N \cap M = N$
and $N \cup M = M$

Complementary Sets

Consider a set A in the universal set U of natural numbers. All elements in U, which are not elements in A, form the complementary set of A. It is written several ways, all having the same meaning.

Three of these ways are $\overline{A}$, A', $\sim A$. All of these are read "the complement of set A." It follows that $U - A = A'$. The expression $U - A = A'$ means that when the elements of set A are removed from set U, the remaining elements form a new set, A', the complement of set A. U - A is not a subtraction problem.

A and A' are disjoint.

$A \cap A' = \emptyset$
$A \cup A = U$

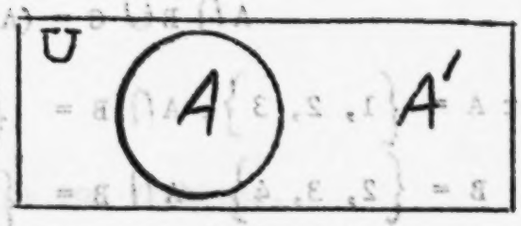


Figure 9

Solving Problems with Sets

The fundamental operations of addition, subtraction, multiplication, and division are used in arithmetic to solve problems and obtain results. The sum or product with respect to the natural number systems is a natural number. Closure is preserved. The fundamental operations used with sets to solve problems and obtain results are union and intersection. The result obtained by the operation of intersection or union of two sets is another set with members in the universal set. Closure is preserved.

In arithmetic, to find the sum of 3 + 4 + 5, the sum of 3 and 4, (7), is found first and then the sum of 7 and 5, (12). Only two numbers can be added at a time. The same is true with multiplication. To find the product of 3 x 4 x 5, the product of 3 and 4, (12), is found first and then the product of 12 x 5, (60). In addition or multiplication, the operation is performed on only two numbers at a time. Addition and multiplication are binary operations.

The same is true when working with sets. If three sets, A, B, C, are involved in an operation or operations, the result with two sets is found first. Then the final answer is found by the operation on the first result and the third set.

$$A \cap B \cup C = (A \cap B) \cup C$$

Let $A = \{1, 2, 3\}$ $A \cap B = \{1, 2, 3\} \cap \{2, 3, 4\}$

$B = \{2, 3, 4\}$ $A \cap B = \{2, 3\}$

$C = \{1, 5, 6\}$ $(A \cap B) \cup C = \{2, 3\} \cup \{1, 5, 6\}$

$$(A \cap B) \cup C = \{1, 2, 3, 5, 6\}$$

Note that the elements 3, 4, 5 are in both sets K and L, but they appear only once in the set formed by the union of sets K and L.

Figure 8 shows the intersection of sets M and N when N is a proper subset of M.

If $N \subset M$,
then $N \cap M = N$
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Figure 8

Complementary Sets

Consider a set A in the universal set U of natural numbers. All elements in U, which are not elements in A, form the complementary set of A. It is written several ways, all having the same meaning.

$$A', \bar{A}, \complement A$$

Three of these ways are A', $\bar{A}$, $\complement A$. All of these are read "the complement of set A." It follows that $U - A = A'$. The expression $U - A = A'$ means that when the elements of set A are removed from set U, the remaining elements form a new set, A', the complement of set A. $U - A$ is not a subtraction problem.

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